HINTS AND ANSWERS.

McLELLAN'S

Elements of Algebra.

HINTS AND ANSWERS

TO THE EXERCISES

IN

ELEMENTS OF ALGEBRA.

BY

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ANSWERS.

EXERCISE XIV [b]. (Page 32.)

1. $6x^2 - 12x^3y + 7xy^2 - 3y^3 - y^2 + 2xy^3.$
2. $-8(m+n) + 5(a+b).$ 3. $0.$ 4. $8(m+n)^2 - y.$
5. $x^3 + x^2y + 7xy^2 + 2y^2 + y^3.$ 6. $\frac{3}{2}a^2 - a^3 - 2a^2b + \frac{1}{4}ab^2 + b^3.$
7. $10x \div y - 13m \div n.$

EXERCISE XV.

1. $(3 + 6b + 7a)x + (-2 - 4)y + m + n.$
2. $(a + m + 1)x + (1 + n - d)y.$
3. $(6a - 3b - 2)x + (1 + \frac{3}{4}b + \frac{2}{3}a)y.$
4. $(2d - 2f)x + (3e - 3d)y + (4f + 4e)z.$
5. $(a + b - 4)x + (\frac{1}{2}a^2 + c^2 - \frac{2}{3}a^3 - 6)y.$ 6. $3ax - 3by.$
7. $(a - 6)x + (5m + 5)\sqrt{y} + (b - 1)y - 3\sqrt{x}.$
8. $(a - c)x^2 + (b - a)y^2 + (c - b)z^2 + ax + by + cz.$
9. $(1 - 9a - 2b - c)x^n + (1 + 7b + 10a - 3abc)y^n.$

EXERCISE XVI [b]. (Page 34.)

1. $-9x^4 + 12x^3y + 6x^2y^2 - 18xy^3 + 21y^4 + 20.$
2. $p^2 - 17q^2 - 22r^2 + 17pq - 8yz^2 + 99.$
3. $-3(x - y) + 34(x - z).$ 4. $19(a - b) - 8(a + b) + 14a + b.$
5. $12\dfrac{x}{y} - 11\dfrac{y}{x} - 8\dfrac{z}{x} - 8\dfrac{a}{b} - 6\dfrac{y}{z}.$
6. $2x^2 - \frac{4}{3}xy - \frac{1}{2}y^2.$ 7. $-\frac{1}{8}a^3 - \frac{2}{3}a^2x - \frac{3}{2}ax^2.$
8. $-2x^3 + 3xy^2 - y^3 - 14x^2 + 2xy - 10y^2 + 2.$
9. $a^2b^2 - 3a^2bc - 3ab^2c - a^2c^2 - abc^2 - b^2c^2.$ 10. $a^4 - 2a^2b^2 + b^4.$
11. $(a^2 - b^2)x^2 + (b^2 + c^2)y^2 + (c^2 - a^2)z^2.$
12. $19a^m - 17b^n - 2c^p + 10d^q.$
13. $(a - p)x^3 + (q - b)x^2 + (1 - r)x + 1.$

14. $a^3 + a^2b + 6ab^2 - 2b^3 + 3b^2$.　　15. $\frac{1}{2}y - 1\frac{3}{6}a - \frac{3}{4}x$.

16. $-\frac{9}{7}(xyz - bx + cy)^2 - 4\frac{3}{10}(z - y + ax)$.

17. *Sum* is $37\frac{3}{4}a - 12\frac{13}{24}b - 4\frac{17}{24}c - 7\frac{1}{6}d - 28\frac{1}{2}e$; the several re-
　　mainders are $35\frac{1}{4}a - 9\frac{7}{24}b - 11\frac{11}{24}c - 1\frac{5}{6}d - 24\frac{1}{4}e$;

　　　$33\frac{11}{12}a - 11\frac{5}{8}b - 5\frac{23}{24}c - 6\frac{5}{12}d - 20\frac{11}{12}e$;

　　　$31\frac{2}{3}a - 12\frac{7}{8}b - 3\frac{5}{24}c - 5\frac{1}{4}d - 22\frac{1}{8}e$;

　　　$29\frac{1}{6}a - 8\frac{1}{8}b + 3\frac{1}{8}c + 1\frac{7}{8}d - 17e$;

　　　$22\frac{5}{12}a - \frac{5}{8}b - 4\frac{1}{4}c - 4\frac{11}{24}d - 12\frac{3}{8}e$;

　　　$16\frac{2}{3}a + 2\frac{7}{8}b - 1\frac{11}{8}c - 1\frac{1}{12}d - 7\frac{1}{8}e$;

　　　$9\frac{1}{3}a + 8\frac{1}{8}b + 4\frac{3}{8}c + 6\frac{2}{3}d - 4\frac{1}{8}e$;　0.

18. i. 0; a; $\sqrt[3]{(3a^3)}$.　　ii. $26a^6$; $a + x$.

EXERCISE XVII [b]. (Page 39.)

1.　6, 18.

2.　30, 40.

3.　35, 65.

4.　4, 2, 10.

5.　8, 40, 12.

6.　500.

7.　120, 137, 163.

8.　120.

9.　30 minutes.

10.　$840.

11.　7, 42.

12.　$32, $36, $44.

13.　24.

14.　$138\frac{8}{9}$, $236\frac{1}{9}$.

15.　£1200.

16.　7 months.

17.　450, 180, 140.

18.　300.

19.　$100.

20.　$12\{22x - \frac{5}{11}(20 - 33x)\} = 44x$, $x = \frac{3}{11}$.

EXERCISE XVIII. (Page 41.)

1.　$a^2 + b^2 - c^2 - d^2$; $a^2 - b^2 + c^2 + d^2$.　　2.　$a^2 - 3b^2 + c^2$.

3.　$2m - n + 6$.　　4.　$-2x - 3y - 2z$.　　5.　$1\frac{1}{3}x - 4\frac{5}{6}y + 1\frac{1}{3}z$.

EXERCISE XIX. (Page 43.)

1.　$-2a + 3x + 3b$.　　2.　$a + b + c$.　　3.　$2ab + 4b^2$.

4.　$-3x - y + 4z$.　　5.　$5 - 4x$.　　6.　$2a - 3b - 3c + 4d$.

7.　$-4a$.　　8.　$-x - 10y + 2z$.　　9.　$-2x + 2y$.

10.　$2x - 6y - my + 4ab - 5$.　　　11.　$3a - 5b - c$.

12.　0.　　13.　$-\frac{1}{6}y$.　　14.　$\frac{4}{5}a - 2b$.　　15.　$\frac{1}{2}x$.　　16.　9.

EXERCISE XX. (Page 44.)

1. i. 　$x - (a + b)$; $x - (a + 3b - 2y)$.

　ii. 　$x - (2m - 2n)$; $x - (3b - 2c - 5d)$.

　iii. 　$x - (2m + 3a - 2b)$; $x - (b - a - c - m + n)$.

　iv. 　$x - (a + b - c - 12)$; $x - \{(a + b) + (p + q) + (m - n)\}$.

2. i. $(2a - 4b - 3c)\,x - (6a + 3c)\,y + (4b - ac)\,z.$

   ii. $(a - b + c)\,x - (a + b - c)\,y - (a - b - c)\,z.$

   iii. $(12a - 15c)\,x - (12a + 4b + 6c)\,y - (12b + 3c)\,z.$

3. i. $2 + (7 - 2c)\,x + (5a - 3)\,x^2 + (9a - 7)\,x^3.$

   ii. $(2c - a^2)\,x^5 + (a - 3b)\,x^4 + (1 - m)\,x^3 + (4c - 3ab)\,x.$

   iii. $(1 - a)\,x^4 + (1 - b + c)\,x^3 + (b - 1)\,x^2 + (a - 7)\,x + 2.$

4. i. $- (3c^2 - 5a)\,x - (abc - 7)\,x^3 - (ab - 7)\,x^5.$

   ii. $1 - (a - 1)\,x - (1 - b)\,x^2 - (a - c + 1)\,x^3 - (a - b - 1)\,x^4.$

   iii. $- (a - 3b^2)\,x^4 - (1 - c)\,x^3 - (1 + 5c^2)\,x^2 - (b + c)\,x.$

5. i. $(a - c + 1)\,x^3 - (a + 2b + 1)\,x^2 + (b + c)\,x + 3.$

   ii. $(5a + 4c)\,x^3 + (7c - 6b + 3a)\,x^2 + (2a - 7b)\,x.$

   iii. $(a - b + c)\,x^2 - 2\,(a + b + c)\,x + ab - bc - ca.$

6. i. $6\,;\ 6.$ ii. $-17\,;\ -9.$ iii. $-1\,;\ -56.$

7. $(a + b + c)\,(x + y + z).$

8. $-3a - rx - (2 - b)\,x^2 + (4a - p - 1)\,x^3 + 2x^4.$

9. $(6y + 1)\,x^5 - (z + 2y)\,x^4 - (2z + 3)\,x.$

EXERCISE XXI [b]. (Page 48.)

1. $36\,;\ -48\,;\ 5\,;\ 9\,;\ -168\,;\ -180.$

2. i. $m^3xyz\,;\ abcx^3\,;\ -24a^3b^3.$ ii. $-36a^6m^4\,;\ -a^3b^3c^3x^2y^2z^3.$

   iii. $-14a^2b^2x^2\,;\ -18x^3y^2z^4\,;\ -5x^3y^3z^2.$

3. i. $40\,;\ -63\,;\ -2\,;\ -37.$ ii. $130\,;\ -880\,;\ 0.$ iii. $\tfrac{1}{2}\,;\ 29.$

EXERCISE XXII [b]. (Page 49.)

1. $a^3b^2c - ab^4 + abc^3\,;\ -\tfrac{5}{2}x^2 + \tfrac{5}{3}xy + \tfrac{10}{3}x.$

2. $3abxy + 6acxz + 15ax\,;\ 9x^4yz^2 - 12x^3y^3z^2 + 15x^2yz^4.$

3. $-15x^5y - 10x^4y^2 + 35x^3y^3 - 5x^2y^4\,;\ 3a^4 + 2a^5b - a^6b^2.$

4. $3x^3y^2z - 3x^2y^4z + 3x^2y^2z^4 - 12x^4y^4z^3\,;\ \tfrac{1}{4}a^2x - \tfrac{1}{16}abx - \tfrac{3}{8}acx.$

5. $-2a^5x^3 + \tfrac{5}{2}a^4x^4 + a^6x^4\,;\ -x^5y^6 + \tfrac{16}{9}x^5y^6.$

6. $\tfrac{3}{4}x^4y^2z^2 - \tfrac{3}{2}x^3y^3z^2 + \tfrac{3}{2}x^2y^4z^2 - x^3y^3z^3\,;\ \tfrac{5}{2}a^4x^2 - \tfrac{5}{3}a^2x^3 + a^2x^4.$

7. $-2\,(a + b)^4 + 2\,(a + b)^2\,;\ -3\,(a - b)^3 - 2\,(a - b)^5.$

8. $(m^2 - n)^7 + (m^2 - n)^5\,;\ 3\,(a + b)^{n-1} + 2\,(a + b)^{n+4}.$

9. $(a + b)^{n+1} + (a + b)^{m+1}\,;\ (a - b)^{n+1} - (a - b)^{n+2}.$

EXERCISE XXIII. (Page 50.)

1. $6x^2 - 13xy + 6y^2$.
2. $15x^3 - 3x^2y - 5bx + by$.
3. $x^3 - 9a^2x$.
4. $-10b^4 - 15ab^3 + 14ab^2 + 21a^2b$.
5. $a^3 + b^3$.
6. $a^2 - b^3$.
7. $a^6 - b^6$.
8. $y^5 - 5y^3 + 2y^2 + 6y - 4$.
9. $a^3 + \frac{17}{3}a^2b + \frac{1}{3}ab - 2ab^2 - \frac{1}{9}b^2$.
10. $(a^2 - b^2)\,x^{n+1}$.
11. $x^5 - a^5$.
12. $1 - x^6$.
13. $y^6 + 2y^4 - 7y^2 - 16$.
14. $\frac{1}{4}x^4 + y^4$.
15. $am + (an - bm)\,x + (ap - bn)\,x^2 - bpx^3$.
16. $a - (a^2 - b)\,x + cx^2 - (ac - b^2)\,x^3 + bcx^4$.
17. $x^8 - 3x^7 - 3x^6 + 6x^5 + 4x^4 - 5x^3 + 6x^2 - 12x + 6$.
18. $a^3 + b^3 - c^3 + 3abc$.
19. $x^3 + y^3 + 3xy - 1$.
20. $18x^9 + 27x^7 + 7x^6 + 3x^5 - 2x^4 + 65x^3 + 115x^2 + 49x + 6$.
21. $(x + y)^2 - (z + a)^2$.
22. $16a^2 + 24ab + 9b^2 - 4c^2 - 4cd - d^2$.
23. $16a^2 - 24ab + 9b^2 - 4c^2 + 4cd - d^2$.
24. $x^4 + 2x^3 + x^2 - y^4 + 2y^3 - y^2$.
25. $a^3 + 8b^3 - 27c^3 + 18abc$.
26. $81x^4 - 256a^4$.
27. $x^6 + 2x^3y^3 + y^6$.
28. $\frac{9}{8}x^4 - \frac{3}{2}ax^3 + \frac{1}{2}a^2x^2 - \frac{2}{9}a^4$.
29. $x^8 - a^8$.
30. $a^2x^{m+2} + abx^{m+2} + abx^{m+3} + b^2x^{n+3} - bx^3 - ax^2$.
31. $x^{2m} + x^{2m}y^m - x^my^m - y^{3m}$.
32. $x^8 - a^8b^8$.
33. $x^{16} - 1$.
34. $x^8 + x^4a^4 + a^8$.
35. $x^4 - y^4 - 4y^3 - 6y^2 - 4y - 1$.

EXERCISE XXIV [a]. (Page 53.)

1. $x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$.
2. $x^6 - 3x^4 + 3x^2 - 1$.
3. $2x^5 - 18x^4 + 39x^3 - 25x^2 + x + 1$.
4. $4x^6 - 5x^5 + 8x^4 - 10x^3 - 8x^2 - 5x - 4$.
5. $21x^8 + 14x^7 - 49x^6 - 8x^5 - 10x^4 + 41x^3 - x^2 - 14x + 2$.
6. $3x^6 + 7x^5 - 12x^4 + 2x^3 - 3x^2 + 13x - 6$.
7. $4x^6 - 8x^5 + 4x^4 - 12x^3 + 12x^2 - 6x + 9$.
8. $x^8 - 3x^6 + 6x^4 - 7x^2 + 3$.
9. $x^6 - 57x^4 + 266x^2 - 1$.
10. $18x^8 + 21x^7 + 8x^6 + x^5 + 63x^3 + 96x^2 + 43x + 6$.
11. $x^6 - 3x^4a^2 + 3x^2a^4 - a^6$.
12. $1 - x^7$.
13. $4 - 12a + 5a^2 + 14a^3 - 11a^4 - 4a^5 + 4a^6$.
14. $1 + x + x^3 + x^4 + x^{17}$.

15. $akx^5 + (al + bk) x^4 + (am + bl + ck) x^3 + (an + bm + cl) x^2$
    $(bn + cm) x + cn.$

[b.]

2. 0.   3. $y^4 - 7y^2 + 10.$ 5. $729x^6 - 117649.$

6. $2a^2 - 2ap - 2a^2n + p^2 + 2anp - 2an + np + 2an^2.$

7. 0, put $a = b + c.$

EXERCISE XXV. (PAGE 56.)

1. $3x; 7x; -3x^2; -5x^2.$ 2. $-a^3c^4; a^4; -7a^2b^2c^2.$

3. $3a^2; -2x; -2x^{m-3}.$ 4. $3a^5b^5c; \frac{7}{4}xy; -\frac{1}{2}x.$

5. $-a^2c; -a^{p-1}; ax^4.$ 6. $-2a^{m-1}b^{n-1}; -a; ma; -2x^{2p+2}.$

7. $3a^{m-n}p^{n-1}; 4a^3(x-y)^{n-3}; -(a+b)^{n-m}.$

8. $-4mx^4 \div 5a^3; -3ab \div 4c; 12a \div c.$

9. $a^2c^2 \div b; -a^{m-1}; x^{n-4} \div y^{n-4}.$ 10. $mx^m \div ny^n; a^2b^2c^2 \div x^3.$

EXERCISE XXVI. (PAGE 57.)

1. $x - 2y; -x^2 + y^2; a^2b - a.$

2. $1 - 3ax - 4a^2x^2; -1 \div x + 2abx.$

3. $-a + b + c; -a + b + b^2; \frac{1}{4}xy - \frac{1}{6}.$

4. $-3mx^{m-n} + 2am^3 - \frac{2}{7}a^4mx^{p-n}.$

5. $(a+b); 4(a-b)^2; a^{n-3} + a^{n-4}.$

6. $-\frac{1}{4}x^3y^3 + \frac{8}{3}x^2y - 2y; 3x^2 - \frac{36}{28}y + 4.$

7. $-a^n - x; a^{2n} - a^nx^n + x^{2n}.$ 8. $6a^2xy - 5ay^2 + 3a^2xy - 4ay^2.$

9. $-\frac{1}{2}x^4 \div y + \frac{1}{4}x^3 - \frac{3}{4}x^2y^2 + \frac{1}{4}x^2.$

10. $\frac{2}{5c^2} - \frac{3}{5a^2} + \frac{4}{5a^2}; 4x^6 - x^2 + \frac{3}{2x^2}.$

11. $-\frac{x}{4y} + \frac{1}{4} - \frac{3}{4y}; \frac{a^3}{x^3} - \frac{a^2}{x^2} + \frac{a}{x} - 1.$

12. $\frac{3x^2}{2a^2} - \frac{5}{2a} + 3 + \frac{a}{2x^2}; (a-b)^{n-1}.$

13. $\frac{3}{2}(a+b)^3 - (a+b) + \frac{1}{2}.$ 14. $a^{m-n} - (a-b)^{m-n}.$

15. $2(x+y)^{m-2}(x-y)^{n-2} - (x+y)^{p-2}(x-y)^{q-2}.$

16. $(a+b)^{m-3}(a-b)^{n-3} - (a+b)^{n-3}(a-b)^{m-3}.$

EXERCISE XXVII. (Page 60.)

1. $x + 7$. 　　2. $a - 6$. 　　3. $3x - 2$. 　　4. $a - 24$.

5. $3x + 1$. 　　6. $3x - 7$. 　　7. $3x + 2$. 　　8. $4x + 3$.

9. $3x - 2y$. 　10. $x - 7$. 　　11. $3x + 4$. 　　12. $5x - 1$.

13. $x^2 - y^2$. 　14. $9x^2 + 4y^2$. 　15. $8x + 3y$. 　16. $x^2 + 14x$.

17. $4x^2 - x$. 　18. $x^2 + 3x + 1$. 　19. $a^2 + a - 1$.

20. $a^2x^2 + ax + 1$. 　21. $2ab$. 　　22. $a^4 - 3ba^2 + 2b^2a$.

23. $x^2 + 2xy + 2y^2$. 　24. $x^2 - 5x + 6$. 　25. $x^2 - 2x + 2$; $-100x$.

26. $24x^2 - 2ax - 35a^2$. 　　27. $x^5 + xy^6 + 7ax$.

28. $-5a^2 + 4bd - 8cf$. 　　29. $3a^2 - 5b^2 + 3c^2$.

30. $3x^2 - x + 2$. 　　31. $a - b - c$. 　　32. $5a^2 + 3x^2$.

33. $p^2q + 4pq^2 + 2q^3$. 　34. $x^2 - mx + m^2 - n$; $(m^2 - m^3)x$.

35. $x + y$; $y^{m+1} + 2xy^m$. 　36. $x^{2n} - 2x^ny^n + y^{2n}$.

37. $ax^2 - bx^2 - a^2x + abx + a^3 - a^2b$. (Read a^3b^2 for a^2b^2 in text.)

EXERCISE XXVIII. (Page 61.)

1. $a^2 + b^2 + c^2 - ab + bc + ca$. 　　2. $x^2 - (a + b)x + ab$.

3. $y^4 - (m - 1)y^3 - (m - n - 1)y^2 - (m - 1)y + 1$.

4. $p^2 + q^2 + r^2 + pq + qr - rp$.

5. $1 - x + 2y + x^2 + 2xy + 4y^2$, $1 + x - 2y + x^2 + 2xy + 4y^2$.

6. $x^2 + y^2 + z^2 + 1$. 　7. $3x^2 - x - 2$, rem $2x+1$. 　8. $2x^2 - x + 1$.

EXERCISE XXIX. (Page 64.)

1. $2x^3 + 3x^2 + 4x + 7$. 　　2. $x^2 - 2x + 4$.

3. $5x^2 - 10x + 2$, $3x^2 - 10x + 1$. 　4. $x^3 - 3x^2 + 3x - 1$.

5. $4x^3 - 3x^2 + 2x + 2$. 　　6. $5x^2 - 12x + 12$, $12x - 72$.

7. $5x^2 + 10x + 5$, $-5x^2 - 10x + 27$. 　8. $10x^3$, $10x^4 - 100$.

9. $1 - 2x + 3x^2 - 4x^3 + 5x^4$; $a^4 + 2a^3 + 3a^2 + 4a + 5$.

10. $x^2 + 2xy + 3y^2$; $m^2 - 2m + 3$.

11. $x^4 + 2x^3 + 3x^2 + 2x + 1$; $a^4 - 2a^3b + 3a^2b^2 - 2ab^3 + b^4$.

12. $3x^4 - 2x^3 - 2x + 3$.

13. $x^4 - 3x^2 - 4x + 15$, $54x^2 - 56x + 27$.

14. $x^5 - 3x^4 - 2x^3 + 2x^2 + 3x - 1$, $5x$.

15. $2x^3 - x^2 - 2x + 4$, $24x^2 - 12x + 10$.

16. $x^4 + 4x^3 + 6x^2 + 9x - 4$, rem. 5. 　17. $2x^2 - 4x + 3$.

18. $x^5 - 2x^6 + 3x^4 - 2x^2 + 1 : x^4 - x^3 + x^2 - x + 1$.

19. $x^2 + (a + b) x + ab$; $x^2 - ax + bx - ab$.   20. $2y^2 - vy - \frac{v}{2}$.

21. $5x^4 - 24x^3 + 99x^2 - 400x + 1601$. rem. $- 6400$.

22. $x^5 - x^4 - x^3 + x^2 + 2x + 1$, rem. 101.

23. $2x^4 - x^3 + 2x^2 - 3x + 1$, rem. 10.

24. $\frac{1}{3}x^5 + \frac{4}{3}x^4 - x^3 + \frac{1}{3}x^2 + \frac{2}{3}$, rem. $- 3x^3 + 21x^2 - 3x + 14$;   take factor 3 out of divisor and divide resulting quotient by 3.

EXERCISE XXX.   (PAGE 69.)

1. 8.   2. 1.   3. $4w - 18$.   4. 3.   5. 2.   6. 1.

7. 2.   8. 4.   9. 12.   10. $(a^2 + ab + b^2) \div 2 (a + 2)$.

11. $(n - b + a^2) \div (a - m + 2ab + c)$.

12. $(bc - ab) \div \{a + c + b^2 + (a - c)^2\}$.

13. $(a^2 + b^2 + ab) \div (a + b)$.   14. $6c \div (30 - 11a + 3b + 2c)$.

15. $x = \frac{1}{2} (a + b)$; write P for $x - a$, and Q for $x - b$, and equation becomes $P^2 \div Q^2 = \{P - (a - b)\} \div \{Q + (a - b)\}$, and on clearing of fractions $P^2 - Q^2$ will prove to be a *factor*: $\therefore P^2 - Q^2 = 0$, $P + Q = 0$, etc.   Or, multiply out.

16. $(c^2 - ab) \div (a + b - 2c)$;
   equation is $(x + a) \div (x + b) = (2x + a + c) \div (2x + b + c)$; complete the divisions, square and transpose;
   $\therefore (c - b) \div (x + b) = (a - b) \div (2x + b + c)$, etc.

17. 19.   18. 9.   19. 9.

20. 72 : remove brackets and combine numerical quantities.

21. $4\frac{7}{9}$.   22. 4.

23. 3 : equation is $\frac{1}{13}x + \frac{7}{18}x + \frac{3}{7}x - \frac{10}{11}x = \frac{8}{13} + \frac{2}{7} + \frac{3}{11} + \frac{1}{6}$; or $\frac{217}{234}x - \frac{37}{77}x = 8051 \div 66 \times 91$;
   i. e. $8051x \div 77 \times 234 = 8051 \div 66 \times 91$, etc.

24. $(2ab^2 - 5a) \div (2a - 2b + 3)$.   25. 8.

EXERCISE XXXI.   (PAGE 72.)

1. 240.   2. 12 miles.   3. $8\frac{4}{7}$ miles.   4. 8 men.

5. $na + nb \div a$.   6. $mn (a - b) \div (mn - m - n)$.

7. Price $= \$(22x - 21z) y \div 20x (x - z)$.   8. 50 gal.

9. One-third.   10. 188 oz.; $\frac{1}{20} (x + 32) = \frac{1}{12} (x - 56)$, where $x = $ wt. of lump.

11. 30 eggs.   12. 4.   13. 40.   14. $785\frac{5}{9}$.

15. 16200, 23000.   16. 63.   17. 30 gal.   18. $1080 \div 251$ miles.

19. B in $ac \div (a - b)$ days, A in $ac \div (c - a + b)$.

20. 22 gals., 9s.   21. $3.   22. 37, 38, 39.   23. 6, 9, 18.   24. 235.

25. Let $x =$ increase of rate, then $c \div a + x = c \div a - b$,
$x = a^2b \div (c - ab)$.

26. 7, 8, 9.   27. $2pqr \div (pq + qr + rp)$.

28. $(ma - b) \div (m - 1)$, $m(b - a) \div (m - 1)$.

29. 432.   30. $n(m - p) \div p$.

31. $20\frac{2}{9}$, $24\frac{2}{9}$. $11\frac{1}{9}$, $44\frac{4}{9}$; if $x =$ 1st part, $x + 4 =$ 2d. $\frac{1}{2}(x + 2) =$ 3d.
$2(x + 2) =$ 4th, and their sum is 100.

EXERCISE XXXII [b].   (PAGE 78.)

1. $169a^2 - 52a + 4a^2$; $225x^2 - 15ax + \frac{1}{4}a^2$;
$441x^2y^2 + 126x^2y + 9x^2$; $144a^2b^4 - 144a^3b^3c + 36a^4b^2c^2$.

2. $\frac{4}{9}x^4 + x^2y^2 + \frac{9}{16}y^4$; $18\frac{1}{16}a^4b^2 - 2a^3b^3 + \frac{16}{289}a^2b^4$;
$289x^4y^6z^8 - 2x^6y^6z^6 + \frac{1}{289}x^8y^6z^4$.

3. $1,024,144$; $1,096,004$; 12321 ; 5625 ; 2401.

4. $25a^{116}b^{14} + 60a^{149}b^{12} + 36a^{182}b^{10}$; $a^{224} - 2a^{112}b^{20} + b^{40}$;
$5929a^{154} + 13552a^{77}b^{88} + 7744b^{176}$.

5. $169x^2 - 4$; $\frac{1}{4}x^2 - \frac{1}{400}$; $4x^4 - \frac{1}{144}y^4$.

6. $49x^{14} - 256x^2y^2$; $\frac{1}{49}x^{14} - \frac{1}{36}x^2y^2$; $x^{154} - y^{176}$.

7. $999,856$; 9879 ; 4875 ; 2499.

8. $x^{224} - b^{400}$; $25a^{116}b^{144} - 36a^{182}b^{114}$; $5929a^{154} - 7744b^{176}$.

9. $4x^2$.   10. $a^2 + b^2 + 2ab - c^2$; $x^2 - 2xy + y^2 - z^2$.

11. $4a^2 - b^2 + 6bc - 9c^2$; $y^2 - 4x^2 + 12xz - 9z^2$.

12. $(w + y)^2 - (x + z)^2$; $(s + t)^2 - (u + r)^2$.

13. $(a + d)^2 - (2b - 3c)^2$; $(3y + z)^2 - (x - 2k)^2$.

14. $2m^2 + 6ms - 8p^2 + 4pk - 12ps - 2mk$.

EXERCISE XXXIII [a].   (PAGE 80.)

6. $1 + 2x + 3x^2 + 2x^3 + x^4$; $1 - 2x + 3x^2 - 2x^3 + x^4$:
$1 + 4x + 6x^2 + 4x^3 + x^4$: $1 - 4x + 6x^2 - 4x^3 + x^4$.

7. $16 + x^2 + 4y^2 + 8x - 16y - 4xy$; $25 + y^2 + 9z^2 - 10y - 30z + 6yz$;
$1 - 2x - x^2 + 2x^3 + x^4$: $x^4 + y^4 + z^4 + 2x^2y^2 + 2y^2z^2 + 2z^2x^2$.

3. $1 + 2x^2 + 6x^3 + x^4 + 6x^5 + 9x^6$;
$1 - 2x^2 + 6x^3 + x^4 - 6x^5 + 9x^6$;
$4 - 4y + 9y^2 - 4y^3 + 4y^4$; $4x^4 + y^2 + 1 + 4x^2y - 2y - 4x^2$.

9. $1 - 2x + 5x^2 - 4x^3 + 4x^4$; $1 + 2x - 5x^2 - 6x^3 + 9x^4$;
$4a^4 - 7a^2 + 4 - 4a^3 + 4a$; $1 + 2a^2 + 2a^3 + 2a^5 + a^9$.

10. $1 + x^2 + b^2y^2 + 2x + 2by + 2bxy$;
$1 + a^2x^2 + b^2y^2 + 2ax + 2by + 2abxy$;
$1 + a^2x^2 + b^2y^2 - 2ax - 2by + 2abxy$;
$1 - 2ax^2 + 2bx^3 + a^2x^4 - 2abx^5 + b^2x^6$.

11. $1 + 2x + 3x^2 + 4x^3 + 3x^4 + 2x^5 + x^6$;
$1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6$;
$1 - 2x - x^2 + 3x^4 + 2x^5 + x^6$.

12. $1 - 4ax + 10a^2x^2 - 12a^3x^3 + 9a^4x^4$;
$x^6 - 6x^5 + 13x^4 - 14x^3 + 10x^2 - 4x + 1$;
$x^6 - 4x^5 + 10x^4 - 4x^3 - 7x^2 + 24x + 16$.

13. $4a^2 + b^2 + 4c^2 - 4ab + 8ac - 4bc$;
$a^2 + \frac{1}{4}b^2 + \frac{1}{4}c^2 - ab + ac - \frac{1}{2}bc$;
$\frac{1}{4}a^2 + \frac{1}{4}b^2 + c^2 - \frac{1}{2}ab - bc + ac$;
$\frac{1}{4}a^2 + b^2 + \frac{1}{9}c^2 - ab - \frac{2}{3}bc + \frac{1}{3}ac$.

[b.]

1. $(2x+y)^2 \div 160$.　　2. $(a+b-c)^2 \div 100$.　　3. $(x+2y+3w+4z)^2$.

4. $6(ab + bc - ca)^2$.　　6. $4(w^2 + x^2 + y^2 + z^2)$.　　7. $8x^3y$.

8. $a^8 + 2a^6 + 3a^4 + 2a^2 + 1$.

10. $2a^2b^2 + 2b^2c^2 + 2c^2a^2 - a^4 - b^4 - c^4$; see Ex. 3, p. 128.

EXERCISE XXXIV [b]. (PAGE 84.)

4. $(3x + 4y)^2 - 25z^2$; $(2a + 4c)^2 - 9b^2$.

5. $\{(x^3 + 2x^2 + 4) - 3x\} \times \{(x^3 + 2x^2 + 4) - 5x\}$
$= x^6 + 4x^5 - 4x^4 - 8x^3 + 31x^2 - 32x + 16$;
$\{(x + z + w) + y\}\{(x + z + w) + 3y\}$
$= (x + z + w)^2 + 4y(x + z + w) + 3y^2$.

6. $x^3 + 9x^2 + 26x + 24$; $x^3 + 14x^2 + 55x + 42$;
$x^3 + 9x^2 + 23x + 15$.

7. $x^3 - 9x^2 + 26x - 24$; $x^3 - 14x^2 + 55x - 42$;
$x^3 - 9x^2 + 23x - 15$.

8. $x^3 + 3x^2 - 10x - 24$; $x^3 - 12x^2 + 29x + 42$;
$x^3 + x^2 - 17x + 15$.

9. $8x^3 + 12x^2 + 22x + 6$; $8x^3 - 12x^2 + 22x - 6$;
$8x^3 - 4x^2 - 10x + 6$.

10. $x^4 + x^3(y + z + w + k) + x^2(wy + wz + wk + yz + yk + zk)$
$+ x(yzw + yzk + zwk + ykw) + yzwk$;
$x^4 - (a + b + c + d)x^3 + (ab + ac + ad + bc + bd + cd)x^2$
$- (abc + abd + acd + bcd)x + abcd$.

11. $w^3 + 3w^2r + 3wr^2 + r^3$; $w^4 + 4w^3r + 6w^2r^2 + 4wr^3 + r^4$;
$8w^3 + 12w^2r + 6wr^2 + r^3$; $w^4 + 8w^3r + 24w^2r^2 + 32wr^3 + 16r^4$;
$w^3 - 3w^2r + 3wr^2 - r^3$; $w^4 - 4w^3r + 6w^2r^2 - 4wr^3 + r^4$.

12. $k^5 + 15k^4s + 90k^3s^2 + 270k^2s^3 + 405ks^4 + 243s^5$;
$a^6 - 12a^5b + 60a^4b^2 - 160a^3b^3 + 240a^2b^4 - 192ab^5 + 64b^6$;
$8a^3 - 6a^2w + \frac{3}{2}aw^2 - \frac{1}{8}w^3$; $\frac{1}{8}a^3 + \frac{3}{2}a^2w + 6aw^2 + 8w^3$;
$27a^6 - 9a^4 + a^2 - \frac{1}{27}$.

13. $1320a^5b^2$; $- 22680a^4b^3$; $- 2a^2x^3$.

14. $1485a^2b^{53} + 55ab^{54} + b^{55}$; $2145x^2y^{64} + 66xy^{65} + y^{66}$;
$- 6655a^2 + 121a - 1$.

15. $54a^2b^2$; $540a^3b^3$; $1680a^4$.   16. 1.21662924 ; 1.7101875.

EXERCISE XXXV [a]. (PAGE 87.)

1. $x^4 + 2x^3 - 85x^2 - 86x + 1680$.

2. Write k for $x + a$, m for $x + b$, $\therefore$ product $= k^4 + k^2m^2 + m^4$
$= 3x^4 + 6x^3(a + b) + x^2(7a^2 + 4ab + 7b^2)$
$+ x(4a^3 + 2a^2b + 2ab^2 + 4b^3) + (a^4 + a^2b^2 + b^4)$.

3. $a^2b^2 + c^2d^2 - a^2c^2 - b^2d^2$.   4. $a^3 + b^3 + c^3 - 3abc$.

5. $x^5 - px^4 + qx^3 - qx^2 + px - 1$.

6. $a^3(x^3 - 1) - a^2(x^3 + x^2 - 2) + a(4x^2 + 3x + 2) - 3(x + 1)$.

7. $w^2 - z^2$.   8. $8x^3$.   9. $24xyz$.   10. $zw + xy$.

11. $6xyz$.   12. $4x^2y^2$.   13. See p. 85, H. (3).

EXERCISE XXXVI [b]. (PAGE 88.)

7. $a + b - c$; $x - 2y - 3z$.   8. $a^2 - 2ab + b^2$; $a^2 + ab + b^2$.

9. $1 + 2x + 3x^2$; $3a^2 + 2a + 3$.   12. $x + 4$; $2x - 3b$.

13. $a + 8b$; $2a - 7b$.   14. $1 + a^2$.   15. $x^2 - 2x + 1$.

[c.]

1. $\dfrac{3a}{5} - \dfrac{5}{3a}$; $\dfrac{a}{2b} - 2$; $\dfrac{8m}{3w} + 2$.

2. $\frac{1}{8}x^2 + \frac{1}{2}x - 1$; $x^2 + x - \frac{1}{2}$; $x^2 + 2x + 1$.

3. $2x + 3y - 5z$; $2x^2 - x + 1$.   4. $\frac{4y}{x} - 4 + \frac{x}{y}$; $x - 2 - \frac{1}{x}$.

5. $\frac{3a}{b} - \frac{1}{5} + \frac{2b}{3a}$.   6. $4a - 3b$; $\frac{2}{x^2} - 3x$.

7. $1 - 2x + 3x^2$; $\frac{1}{2}x - 1$.   8. $a + 2b - c$.   9. $\frac{x}{3} - 1 + \frac{3}{x}$; $\frac{x}{3} + 2$.

10. Cube both sides by formula G (2), p. 85.

EXERCISE XXXVII.

1. $a^2b + b^2a$; $a(a+b)^2 + b(b+a)^2$;
    $ab(b-c) + bc(c-a) + ca(a-b)$; $a^2bc + b^2ca + c^2ab$;
    $a(b+c) + b(c+a) + c(a+b)$.

2. $(a-b)(b-c) + (b-c)(c-a) + (c-a)(a-b)$;
    $a^2(b-c) + b^2(c-a) + c^2(a-b)$;
    $a(b-c)^2 + b(c-a)^2 + c(a-b)^2$;
    $(x-a)(b-c)^2 + (x-b)(c-a)^2 + (x-c)(a-b)^2$.

3. $a^3 + b^3 + c^3 + d^3$;
    $a^2(bc + bd + cd) + b^2(ac + ad + cd)$
    $+ c^2(ab + ad + bc) + d^2(ab + bc + ac)$;
    $a^2(b+c+d) + b^2(c+d+a) + c^2(d+a+b) + d^2(a+b+c)$;
    $a + b + a + c + a + d + b + c + b + d + c + d$;
    $ab + ac + ad + bc + bd + cd$;
    $a^2(a-b) + b^2(b-c) + c^2(c-d) + d^2(d-a)$;
    $(a-b)^3 + (a-c)^3 + (a-d)^3 + (b-c)^3 + (b-d)^3 + (c-d)^3$.

4. $(a-b)^2(b-c)^2 + (a-c)^2(c-d)^2 + (a-d)^2(b-d)^2 + (b-c)^2(c-d)^2$;
    $(x-a)(b-c)^2 + (x-b)(c-d)^2 + (x-c)(d-a)^2 + (x-d)(a-b)^2$.

13. $a, b, -c$; $a, -b, c$; $a, -b, -c$.   14. a, b, c ; a, b.

15. a, b, c ; a, b ; $a, -b$.   16. a, b, c ; ax and by, x and y.

17. a and b ; a, b, c.   18. a, b, c.   19. a, b ; a, b.

20. a, b, c.   21. a^2b.   22. a^4, a^2b, a^2b^2, abc^2.

23. x^3, x^2y, xyz.   24. a^3b.   25. ab^3, ab^2c.

26. xy^2, xyz.   27. x^3, x^2y ; x^4, x^3y, x^2y^2 ; x^5, x^4y, y^3y^2.

28. x^3, x^2y, xyz.   29. a^4, a^2b, a^2b^2, a^2bc ; $x^5, x^4y, x^3yz, x^2y^2z, x^3z^2$.

30. a^3, a^2b, abc ; $x^5, x^6y, x^3y^2, x^4y^3$.

31. $a^3 + b^3 + c^3 + d^3 = 3(abc + abd + bcd + cda)$.

32. $a + b - c$; $a - b + c$; $- a + b + c$; $a - b - c$.

33. $\frac{1}{2}\{(a - b)^2 + (b + c)^2 + (c + a)^2\}$;
    $\frac{1}{2}\{(a + b)^2 + (b + c)^2 + (c - a)^2\}$;
    $\frac{1}{2}\{(a + b)^2 + (b - c)^2 + (c + a)^2\}$;
    $\frac{1}{2}\{(a + b)^2 + (b - c)^2 + (c + a)^2\}$;
    the three expressions are derived from the *first* by, respectively, substituting $- c$ for c, $- b$ for b, $- a$ for a : observe, also, that $(- a - c)^2 = + (a + c)^2$.

EXERCISE XXXVIII.   (Page 96.)

1. $3(a^2 + b^2 + c^2) - 2(ab + \&c.)$.   2. 0.   3. $2(xy + yz + zx)$.

4. $6(a^2 + b^2 + c^2) - 2(ab + bc + ca)$.

5. $2(x^2 + y^2 + z^2 - yx - yz - zx)$.

6. $14(a^2 + b^2 + c^2) - 14(ab + bc + ca)$.

7. $4(a^2 + b^2 + c^2 + d^2)$.   8. $4(a^2x^2 + b^2y^2 + c^2z^2)$.

9. $2(a^3 + b^3 + c^3) + 6(a^2b + \text{etc.}) - 12abc$.

10. $a^2 + b^2 + c^2 + d^2$.   11. $3(a^2 + b^2 + c^2 + d^2) + 2(ab + \text{etc.})$.

12. 0.   13. $6abc$.   14. $abc(a + b + c)$.

15. $4(x^4 + y^4 + z^4) + 24(a^2b^2 + b^2c^2 + c^2a^2)$.

16. Note.—The first term in each of the binomial *factors* should have *index* 2 ; *i.e.*, a^2 for a, etc.   Multiply out, or use identity,
    $$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx).$$

17. Multiply out and subs. for *s*.

18. $rs = (a + b)^2 - (c - d)^2$, the other pairs by symmetry ; result is $4(ab + ac + ad + bc + bd + cd)$.

20. Type terms are a^4, $2a^3(b + c)$, a^2b^2, and both expressions reduce to same form.   *Or*, use identity, Ex. 7, p. 105, putting $a - b$ for a, $b - c$ for b, and $\therefore a - c$ for $a + b$.

EXERCISE XL.   (Page 98.)

1. $(a - b)(x + 2y)$.

2. $(a + b)(2x - 3y)$.

3. $(a + x)(a - b)$.

4. $(c - d)(ab - c)$.

5. $(m + n)(x^2 - a)$.

6. $(a + b)(a - c)$.

7. $(a + b)(3x + y)$.

8. $(a - bc)(1 - x)$.

9. $(a - b)(c + y)$.

10. $(a + x)(a + b)$.

11. $(3a - b)(x - y)$.

12. $(7 - x)(a - bc)$.

13. $(r - s)(3p + q)$.   14. $(1 - a)(1 - b)$.

15. $(3x - a)(2x + y)$.   16. $(a^2 - 1)(a + 1)$.

17. $(x - 1)(3bx - 1)$.   18. $(xy - z)(a + bc)$.

19. $(a - 1)(a^2 + 1)$.   20. $(x + f)(2a + b)$.

21. $(x^2 + a^2)(a - 3c)$.   22. $(x - y)(x - 3)$.

23. $(x^2 - 1)(2a^2 - 1)$.   24. $(b - 1)(c - 1)$.

25. $(a^2x^2 - c)(a^2x^2 - b)$.   26. $(3b^2 - 1)(1 - 3a^2)$.

27. $(x^2 - a^2)(x^2 + ax + a^2)$.   28. $(a - b)(x - y + z)$.

29. $(a + b)(ax + by + c)$.   30. $(ax^n - b)(bx^n + a)$.

31. $(a - 1)(a + b)$.   32. $(2 + x^n)(3 - y^n)$.

33. $(1 - b)(a - b + c)$.   34. $(a - x)(2pq - 3bf)$.

35. $(1 - x^2)(1 + x^2 + p + q)$.   36. $(2p^n - 3q^n)(r^n - 2s^n)$.

37. $(a + b - c)(d - e + f)$.   38. $(1 + p + q)(1 - a + b)$.

EXERCISE XLI. (PAGE 100.)

11. $\left(\dfrac{a}{b} - \dfrac{b}{a}\right)^2$; $(1 - x^m)^2$; $\{(2x - 3y) - (2x + 3y)\}^2$.

12. $\{(a^2 + ab + b^2) - (a^2 - ab + b^2)\}^2$; $\left\{\left(\dfrac{x}{y}\right)^m - \left(\dfrac{y}{x}\right)^m\right\}^2$.

13. $(x + y + z)^2$; $(p - q + r)^2$.

14. $(a - 2b + 3c)^2$; $(1 - x + y)^2$.

15. $(3a + 2b + c)^2$; $(2a^2 - 3a + 4)^2$.   16. $(3ax + 2by + cz)^2$.

17. $(2a^2 - 3b + 4c)^2$; $(a^2 - b^2 - c^2)^2$.

18. $\pm 4xy$; $\pm xy$; x^2y^2; $-10xy$; $\pm 4x^2y^2$.

19. $\pm 6ay$; $\pm 10a^3b$; $\pm 12x^3y^2$; $\pm 2a^nb^n$; a^2.

20. a^4; z^2; $\frac{1}{4}$; $\frac{1}{4}$; 4; b^2.   21. $\frac{1}{4}b^2$; $\pm 4bxy^2$; ± 2; ± 2; x^4.

22. 7; $\frac{1}{4}$; $b^2 \div 4a^2$; $25 \div 4$; $49 \div 4$.

23. $81 \div 16$; $x^2 + 4$; $x^2 + 13$; $-c + \frac{1}{4}b^2$.

EXERCISE XLII [b]. (PAGE 103.)

NOTE.—The two factors in each case are expressed with the double sign $\pm$.

1. $a + b \pm c$; $2(x + y) \pm z$; $x \pm (y + z)$; $2 \pm (a + b)$.

2. $p + 2q \pm r$; $4x \pm (a + 3b)$; $2m \pm (p - q)$; $2x(-4y)$.

3. $1 \pm (b - c)$; $a + b + c \pm x$; $(8 + x)(10 - x)$; $b - c \pm (a-x)$.

4. $3\{2(a^2 - bc) \pm (b^2 - ac)\}$; $a - 5b \pm 1$; $1 \pm (x - y + z)$; $(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$; $(a - 3c)(a + 4b + 3c)$.

5. $(-a + b - 4c)(3a - 5b + 4c)$;
$(1 - a + b)(1 - a - b)(1 + 2a - a^2 + b^2)$;
$(12x - 1)(2x + 7)$.

6. $(x - z \pm y)(x + z \pm y)$; $4(x + z)(y + u)$;
$\{x \pm (y + z)\}\{x \pm (y - z)\}$.

7. $(x - z) \pm (y - u)$; $a \pm (x - y)$; $x \pm (y + z)$.

8. $x \pm (y - z)$; $x \pm (y + z)$; $x + z \pm y$; $x^2 \pm (x - 1)$.

9. $(x + a) \pm (y + z)$; $(a - c) \pm (b - d)$;
$(a^8 + b^8)(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$;
$(a^2 + 6a + 5)(a^2 + 2a + 3)$.

10. $a - b \pm (x - y)$; $a^2 + a \pm (b^2 - b)$.

11. $(x + b)(a \pm x)$; $\{a - d \pm (b - c)\}$; $ab \pm c(a - b)$.

12. $\{c \pm (a - b)\}\{a + b \pm c\}$; $x^2 + y^2 \pm (z^2 + 1)$; $a - d \pm (b - c)$.

13. $2a \pm (b - 3c)$; $b \pm (2a - 3c)$; $2a \pm (b + 3c)$.

14. $3c \pm (2a - b)$; $(a + c) \pm (b + d)$; $(a + d) \pm (b + c)$.

15. $(b + c) \pm (a + d)$; $(a + d) \pm (2b - 3c)$.

16. $3c + d \pm (a - 2b)$; $(a - 3c) \pm (2b - d)$.

17. $\{a + d \pm (b - c)\}\{b + c \pm (a - d)\}$.

18. $(x^2 + 1 \div y^2)(x + 1 \div y)(x - 1 \div y)$; $x^4 \pm \frac{1}{16}$, etc.;
$x^3(x^4 - 25) - \frac{1}{4}(x^4 - 25) = (x^4 - 25)(x^3 - \frac{1}{4})$, etc.;
$(x^4 - 16)(x^3 + 1)$, etc.

EXERCISE XLIII. (Page 105.)

1. $3x^2 + y^2 \pm xy$.

2. $4a^2 - b^2 \pm 3ab$.

3. $3a^2 + b^2 \pm 5ab$.

4. $5m^2 + 4n^2 \pm 7mn$.

5. $x^2 + 1 \pm x$.

6. $x^2 + 4 \pm 2x$.

7. $x^2 + 25 \pm 5x$.

8. $x^2 + \frac{9}{4} \pm \frac{3}{2}a$.

9. $x^2 - y^2 \pm 3xy$.

10. $x^2 + \frac{16}{9} \pm \frac{4}{3}x$.

11. $a^2 - y^2 \pm 2ay$.

12. $m^4 - n^4 \pm 4mn$.

13. $9a^2 + b^2 \pm 3ab$.

14. $4a^2 + b^2 \pm 6ab$.

15. $5p^2 - 4q^2 \pm pq$.

16. $9x^2 - y^2 \pm 4xy$.

17. $2x^2 - 1 \pm 2x$.

18. $\frac{1}{2}r^2 + y^2 \pm xy$.

19. $x^2 + 2a^2y^2 \pm 2axy$.

20. $2a^2 + y^2 \pm \frac{7}{2}ay$.

21. $x^4 + y^4 \pm x^2y^2$, etc.; $x^4 + 1 \pm x^2$.

22. $a^2x^4 + 1 \pm ax^2$; $x^4 + 2y^4 \pm 2xy$.

23. $(a + b)^2 + c^2 \pm 3c(a + b)$; $1 + 2x^2 \pm 2x$.

24. $4x^2 + 2(y - z)^2 \pm 5x(y - z)$; $1 + 5z^4 \pm 3z^2$.

25. $1 + 2a^4 \pm 2a^2$; $a^2 + 9b^2 \pm 9ab$.

26. $2(1 + a + a^2)^2$; $x^2 + 1 \div y^2 + x \div y$.

27. $x^2 + 1 \div 2y^2 \pm x \div y$; $a^2 + 2 \div a^2 \pm 2$;
$(a + b)^2 + (a - b)^2 \pm (a^2 - b^2)$.

28. $c^2 + 2(a + b)^2 \pm 2c(a + b)$; $1 \div a^2 + 1 \div b^2 \pm 1 \div ab$;
$3 \div a^2 + 1 \div b^2 \pm 3 \div ab$.

EXERCISE XLIV [a]. (PAGE 108.)

17. $(m^3 + 21)(m^3 + 19)$.   18. $(a^2x + 39)(a^2x + 1)$.

19. $(x^n + 7)(x^n + 12)$.   20. $(x + 17)(x + 23)$.

21. $(x^n + 12)(x^n + 4)$.   22. $(x + 33)(x + 27)$.

23. $(a + 27)(a + 13)$.   24. $(a + 18b)^2$.

25. $(a^2x + 81)^2$.   26. $(x - 4)^2$.   27. $(x - 15)^2$.

28. $(x - 19)^2$.   29. $(x - 20)^2$.   30. $(x - 50)^2$.

31. $(x^3 - 5)(x^3 - 25)$.   32. $(m - 17n)(m - 5n)$.

33. $(x - 13y)^2$.   34. $(x^2 - 5y^2)(x^2 - 4y^2)$.

35. $(a - 27b)(a - 2b)$.   36. $(4 - x)(3 - x)$.

37. $(26 - ab)(5 - ab)$.   38. $(a - 25)(a - 15)$.

39. $x^2 + 1 \pm \frac{10}{3}x$.   40. $(x^3 - 27)(x^3 - 8)$.

41. $3x(x - 2)(x - 8)$.   42. $a(x - 5)(x - 6)$.

43. $x^2 + 60 \pm 17x$.   44. $(x^n - 7)(x^n - 37)$.

45. $(a + b - 4)(a + b - 3)$.   46. $(13 - ax)(11 - ax)$.

47. $(1 - 8x^2y^2)(1 - 51x^2y^2)$.   48. $(a - 27b)^2$.

49. $x^2(a - 15bx)(a - 5bx)$.   50. $(m - 19)^2$.

51. $(p - 27q)^2$.   52. $\{(x - y)^n - 33\}\{(x - y)^n - 11\}$.

[b.] (PAGE 109.)

1. $(a^2 + 1)(a^2 - 2)$.   2. $(a + 3)(a - 2)$.

3. $(x - 3)(x + 2)$.   4. $(x - 16)(x + 3)$.

5. $(x + 12)(x - 7)$.   6. $(y + 12)(y - 5)$.

7. $(a + 20)(a - 7)$.

8. $(a + 25b)(a - 12b)$.

9. $(x + 12)(x - 11)$.

10. $(x - 10)(x + 2)$.

11. $(y^2 - 10a^2)(y^2 + 5a^2)$.

12. $(ab - 4)(ab + 1)$.

13. $3(az^2 - 14)(az^2 + 1)$.

14. $(a^4 - 20)(a^4 + 5)$.

15. $(abc + 11)(abc - 2)$.

16. $(a^2b^2 - 30)(a^2b^2 + 3)$.

17. $(x^2 - 48)(x^2 + 8)$.

18. $(x^n - 16)(x^n + 3)$.

19. $(x + y - 19)(x + y + 18)$.

20. $\{a - 30(b + c)\}\{a + 12(b + c)\}$.

21. $(x^{2n} + 4)(x^{2n} - 3)$.

22. $(20 + a)(19 - a)$.

23. $(13 - ab)(5 + ab)$.

24. $(12 - m)(17 + m)$.

25. $3y(a + 14bx)(a - 2bx)$.

26. $(2x + 7)(2x + 5)$.

27. $(3x + 7)(3x + 5)$.

28. $(2x^2y - 7z^2)(2x^2y + 6z^2)$.

29. $(7a - 8b)^2$.

30. $x(2b - y)(2b - 5y)$.

31. $x^4(8y^3 - 10z)^2$.   32. $(a^2 - 40b^2)(a^2 + 5b^2)$.   33. $(11x^2 - 13y^2)^2$.

34. $3(x^2 + y^2)(3x^2 - 4y^2)$; where $x = a - b$ and $y = c$.

35. $(8x^n - 2b^n)(8x^n + b^n)$.

36. $(\frac{1}{2}x^2 + 7)(\frac{1}{2}x^2 - 6)$.

37. $(\frac{3}{4}a + 7b)(\frac{3}{4}a - 8b)$.

38. $\left(\dfrac{3}{x} - 7\right)\left(\dfrac{3}{x} + 9\right)$.

39. $\left(\dfrac{4}{x} + 19\right)\left(\dfrac{4}{x} - 20\right)$.

40. $(5x^2 + 21)(5x^2 - 31)$.

EXERCISE XLV. (PAGE 112.)

1. $(2x + 1)(2x + 3)$.

2. $(4x + 1)(x + 3)$.

3. $(3x + 3)(5x + 4)$.

4. $(3x + 2)(2x + 1)$.

5. $(2x + 5)(3x + 4)$.

6. $(2x + 7y)(4x + 3y)$.

7. $(4a + 9)(a + 1)$.

8. $(1 + m)(7 + 3m)$.

9. $(x + 5)(4x + 3)$.

10. $(x + 7)(3x + 2)$.

11. $(4x - 3)(3x + 2)$.

12. $(4x + 3)(3x - 2)$.

13. $(4x + 7)(3x - 5)$.

14. $(4x - 7)(3x + 5)$.

15. $(3x + 2)(2x - 1)$.

16. $(5x - 1)(2x - 3)$.

17. $(3x + 4)(5x - 2)$.

18. $(x - 7)(7x - 1)$.

19. $(5x + 2y)(3x - 5y)$.

20. $(a^2 - 19)(a^2 + 17)$.

21. $(3m + 20)(2m - 19)$.

22. $(2a + 20)(3a - 19)$.

23. $(3x + 7y)(4x - 5y)$.

24. $(3 - 12x)(5 + 11x)$.

25. $(5x^2 - 1)(4x^2 + 1)$.

26. $(15a - 1)(a + 15)$.

27. $(12x - 7)(2x + 3)$.

28. $(6 - y)(3 - 5y)$.

29. $(8a + b)(3a - 4b)$.

30. $(8 - 9y)(3 + 8y)$.

31. $(28x^2 - 25)(x^2 + 5)$.
32. $4(14x + 5y)(x - y)$.

33. $4(7x - 5y)(2x - y)$.
34. $4(14x - 5y)(x + y)$.

35. $(8a - 5b)(7a - 4b)$.
36. $2(28y + 1)(y - 10)$.

37. $(8y + 5z)(9y - 8z)$.
38. $(9y + 3a)(4y - 5a)$.

39. $(56a^2 + 4b^2)(a^2 - 5b^2)$.
40. $(56a - 5b)(a - 4b)$.

41. $(13x + 12y)(3x - 4y)$.
42. $(3x + y)(13x - 11y)$.

43. $(39x - 26)(x + 1)$.
44. $(12x + 13y)(5x - 8y)$.

45. $(1 - 13x^2)(1 + 11x^2)$.
46. $(a^n - 13b^n)(a^n + 11b^n)$.

47. $(3x^3 - 21)(4x^3 + 11)$.
48. $(17x - 1)(x + 17)$.

EXERCISE XLVI. (Page 114)

1. $(x - a)(x^2 - 2x - 1)$.
2. $(x - a)(x^2 - px + q)$.

3. $(my - n)(ay^2 + by - c)$.
4. $(2b - c)(x^2 - 2bx + b)$.

5. $(nx - a)(x^2 - x - 1)$.

6. $(bx - a)\{(m + 1)b^2x^2 + (m + 1)(n + 1)abx + (n + 1)a^2\}$;
   multiply out, take m-terms for one group, etc.

7. $(y - b)(y - a)^2$.
8. $(x - b)(x - a)(x + 2b)$.

9. $(x+p+q)(x+q-p)(x-2q)$.
10. $(x - a)(x + b)(x + 3)$.

11. $(x+b)\{x(x-1)-a(x+1)\}$.
12. $(2x - a)(2x^2 + 4x - 3)$.

13. $(py + q)(y^2 - y + 1)$.
14. $(mx - n)(px^2 + qx - r)$.

15. $(mx - n)(ax^2 - cx - b)$.
16. $(px - q)(3x^2 - cx - b)$.

17. $(x^2 - px + q)(ax^2 + bx - c)$.
18. $(2x + 3c)(x^2 + ax - 2b)$.

19. $(2x + 3c)(x^2 - 2ax + 3b)$.
20. $(ap - bq)(2p^2 + 3pq + q^2)$.

21. $(ap - bq)(3p^2 - pq - 2q^2)$.
22. $(ax + b)(cx^2 + dx + c)$.

23. $(ax + b)(2cx^2 - dx - 3c)$.
24. $(3y - ab)(3y - bc)(3y + 5)$.

EXERCISE XLVII. (Page 118.)

1. $(6x - y + 1)(x - 6y - 1)$.
2. $(3x + 2y + 1)(2x - 3y - 1)$.

3. $(4a + 5b + 4)(3a - 4b - 5)$.
4. $(x - y + 3z)(x + 2y - z)$.

5. $(3x + y + 3)(x - 3y + 9)$.
6. $(2a - 5b + 6c)(3a + 4b - 8c)$.

7. $(3a - b - 7)(4a - 3b + 8)$.
8. $(7x - y - 1)(x - y + 3)$.

9. $(a + 3y)(a - 4y - 5)$.
10. $(2x - 5y - 7z)(2x + 3y + 3z)$.

11. $(3x + y - 4z)(3x - 3y - 2z)$.
12. $(3x - 2y + 3z)(2x - 3y + 2z)$.

13. $(5x - 3y + 2z)(x - y - z)$.
14. $(a - 2b + 3c)(11a - b - c)$.

15. $(2a - b - 3c)(4a - 3b - c)$.
16. $(1 - 3x + 4y)(1 + 7x - 5y)$.

EXERCISE XLVIII. (Page 122.)

7. 8.   8. $-8a^3$.   9. -205.   10. 1.

11. $a^3 + pa^2 + qa + r$.   12. -36.   13. 1555.

14. $x + 2,\ x - 3$.   15. $(x + 1)(3x + 2)(2x - 1)$.

16. Last term should be $52a^3$.   18. $(x - 2)(x - 5)(x + 7)$.

19. -535.   20. -800.   21. 101.   22. 115.

23. $-a^3 - pa^2 - qa - r$.   24. $abc - 4ab(a + b)$.

29. -1.   32. 2.   34. $2(a + b)^3$.   35. 0.   39. 0.

40. 0.   41. yes ; put 1 for $x + y$.   42. 0.

44. $a^2 + pa + q,\ a^2 + p'a + q'$.

EXERCISE XLIX. (Page 126.)

5. $(p - 1)^2 - (p - 1)(q + 1) + (q + 1)^2$; $a^n - b^n$.

6. $x^{10} - x^5 a^5 + a^{10}$; $1 - (a - b) + (a - b)^2 - (a - b)^3$.

7. $x^2 - 1 + 1 \div x^2$; $x^8 - 2x^6 + 3x^4 - 2x^2 + 1$.

8. $x^2 + y^2 \pm xy$; $(a^2 + 4b^2)(a + 2b)(a - 2b)$; $2x(x^2 + 12y^2)$.

9. $(a + b)(a^2 + b^2 \pm ab)$;
$x^2 - 2y^2$ and $x^8 + 2x^6 y^2 + 4x^4 y^4 + 8x^2 y^6 + 16y^8$.

10. $(a^2 + bc)(a^4 + 7b^2 c^2 - 4a^2 bc)$; $(x + 1)^n (x - 1)^n$;
$(x + 1)(x - 1)(x^2 + 1)(x^4 + 1)$.

11. $a^3 - (2b^5)^3 = \{a - 2b^5\}\{a^2 + 2ab^5 + 4b^{10}\}$; $(a - b)ab$.

12. Expression $= (4a^2 - 9b^2)(x^3 - 8a^3)$, etc. ; $\left(a - \dfrac{1}{2a}\right)\left(a^2 - 1 + \dfrac{1}{4a^2}\right)$.

15. Expression $= (x^2 - y^2)(x - y)^2 = 128a^3 b^3 (a^2 + b^2)$.

16. $x - 1$, factor dividend.

17. $(a^4 - a^2 x^2 + x^4)(a^8 + a^4 x^4 + x^8)$.

19. Factor and divide by $a + 1 \div a$, $\therefore a^2 - 1 + 1 \div a^2 = 0$,
$\therefore a^2 + \dfrac{1}{a^2} = 1$, $\therefore a^2 + \dfrac{1}{a^2} + 2 = 3$, etc.

20. Expression $= (1 - x)(1 - x^n)$.

21. Divisor $= (x - 1)(x^2 - x + 1)$ and given expression vanishes for each of these factors.

22. $(x - y)(x^2 + y^2)(x^4 + y^4)$.

EXERCISE L [a].   (PAGE 129.)

1. $3(x + y)(y + z)(z + x)$.   2. $(a - b)(b - c)(a - c)$.
3. $(a - b)(b - c)(a - c)$.   4. $(a - b)(b - c)(c - a)$.
5. $3(x - y)(y - z)(z - x)$.   6. $3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$.
7. $(x + y)(y + z)(z + x)$.
8. $-(a - b)(b - c)(c - a)(a + b + c)$.
9. $(a - b)(b - c)(c - a)(a + b + c)$.
10. $(x + y)(y + z)(z + x)$.   11. $3(a^3 - b)(b^3 - c)(c^3 - a)$.
12. $-(a - b)(b - c)(c - a)(a^2 + b^2 + c^2 + ab + bc + ca)$.
13. $(a + b + c)(a^2 + b^2 + c^2)$. 14. $(x + y + z)^3$.   15. $(a+b+c)^3$.
16. $6abc$.   Insert in text $-(a + b)^3$, and read $-$ before $(c + a)^3$.
17. $(a - b)(b - c)(c - a)(ab + bc + ca)$.

[b.]   (PAGE 130.)

2. By symmetry; or formula (II) (4), p. 85.
3. By symmetry; or transpose $3abc$, then a is a factor, etc.
4. $5(x + y)(y + z)(z + x)(x^2 + y^2 + z^2 + xy + yz + zx)$.
5. $(a + b - c)(a^2 + b^2 + c^2 - ab + bc + ca)$;
    $(-a + b + c)(a^2 + b^2 + c^2 + ab - bc + ca)$;
    $(-a - b + c)(a^2 + b^2 + c^2 - ab + bc + ca)$.
6. $12\frac{4}{7}$; 2d term should be $13x^3$.
7. Use synthetic division; $4m - 12 = 0$, $m = 3$.
8. Given expression $= (x - 3)(x + 2)(x - 5)$, which is true for
    all values of x, $\therefore$ coefficient of *like powers* of x are equal;
    i. e., $a = -6$, $b = 1$, $c = 30$.
9. $b = -6$, $c = 5$, $a = 12$.   11. $1 \div (a + b + c)$.
12. 2; dividend is 2 (divisor).   13. $16abc(b - c)(a - c)(a - b)$.
15. $-abc(b - c)(c - a)(a - b)$.
16. $-abc(a + b + c)(b - c)(c - a)(a - b)$.

EXERCISE LII [a].   (PAGE 133.)

1. ax; $x + 2$; $2(x - y)^2$.   2. $2(x - a)$; $ab(x - a)(x - b)$.
3. $x + 3$; $x + 9$.   4. $x - 2$; $x + 2$.
5. $a + b$; $x + 1$; $x - 3$.   6. $x + 5$; $x + 4$; $(x + 1)^2$.
7. $x + 3$; $x - 11y$.   8. $3(x + 3y)$; $(x + y)^2$; $x^2 + 4$.

9. $x + 2$; $x + a$.

10. $x - a$: $x - y$.

11. $x + 3z$; $a + 3$.

12. $a - 1$: $x - a - 4$.

13. $a + b - c$; $a + b + c$.

14. $a + b + x + y$: $x + a$.

15. $x - a$: $8 (x - 3y)$: $x + a$. 16. $x + a$; $x - 5$; $(1 - x)^2$.

17. $x^2 + xy + y^2$: $x^2 + a^2$: $x^2 - y^2$.

18. $3 (x - y) (x + y)$: $3 (a + b) (a^2 + b^2)$.

19. $5 (p - q) (p + q)$; $x + y$.

[b.]

20. $x + y$.

21. $3x + 1$: $5x - 1$.

22. $2a + 5$; $a + 5$.

23. $x + 3$: $(x - 1)^2$.

24. $a^2 + ab + b^2$; $a + b$.

25. $2x + 1$: $x^2 + y$.

26. $x^2 (3x + 2)$.

27. $x^2 + 2y^2 - 2xy$.

28. $3x + 4a$; unity.

29. 4 ; $1 + \dfrac{1}{a}$.

30. $a^2 + \dfrac{1}{a^2}$.

31. $2x - 1$.

32. $2a + 3b - c$.

33. $5 (x + 2y)$.

34. $mx + m - x$.

35. $ap - bq$.

36. $x + 2ab$; omit a in ax^3.

37. $(a - b) (x + a)$.

38. $3 (2a - 7)$.

39. $(2ax - y)$; last term of 2nd expression should be $3y^3$.

40. $(x - 1)^2$.

EXERCISE LIII [a]. (PAGE 141.)

1. $2 (x + 1)^2$.

2. $x - 5$.

3. $2x^2 - 3x - 1$.

4. $7a^2 + 3a - 1$.

5. $y^2 + 8y - 2$.

6. $y^2 - 3y - 5$.

7. $x^2 - 2x - 3$.

8. $x^2 - 3$.

9. $5x^2 - 1$.

10. $3x^2 - 2xy + y^2$. 11. $x - 1$.

12. $(x + 2)^2$.

13. $x (2x^2 + 2xy - y^2)$.

14. $x - 2$.

15. $(x - 1) (x + 1)$ or $x^2 - 1$.

[b.]

1. $x^2 - 3x + 2$.

2. $x^2 - 13x + 5$.

3. $a^2 - 8$.

4. $(x - 3)$.

5. $12x + 5$.

6. $2x^3 - 4x^2 + x - 1$.

7. $x^2 + 3x + 5$.   8. $(x + 1)(x^2 + 1)$.

9. $a^3 - a^3 - a - 1$.   10. $2y^2 - 7$.

11. $2x^2 - 3x - 1$.   12. $x^3 + x^2 - 5x + 3$.

13. $3x - 5a$.   14. $2x^2(2x + 9)$.

15. $x^2 - 7x - 3$.

EXERCISE LIV [*a*]. (Page 143.)

1. $2 \cdot 2a^3$; $2x^3 \cdot 6yz$; $ab \cdot cx^2y$; $a \cdot 12ab^2$; $4z \cdot 7x^2yz$.

2. $42a^3b^3$; $4y^2 \cdot x^4$; $y \cdot 10x^4y^3$; $3c \cdot 7a^2b^3c^2$.

3. $y \cdot x^3y^2$; $2 \cdot 3a^2b^3$; $7a^2 \cdot 3b^2 \cdot 2$; $ab \cdot c$.

4. $2y \cdot 3x^2y \cdot 2$; $8y^3 \cdot 7x^4y$; $2 \cdot 3x^2y^2 \cdot 2xy$; $ax^2 \cdot 6xy$.

5. $2 \cdot 3abc \cdot 2$; $3 \cdot 4x^2y \cdot 5y$; $p \cdot 6pq^2 \cdot p$; $ax^n \cdot by^n$.

6. $1 \cdot a^3b^2c^4$; $xy^3 \cdot 15a^3x^2y$; $n^2 \cdot 3m^3p^4$; p^nq^n.

7. $3x(a - x)$; $3a^2b(a + b)$; $a(a - b^2)$; $abc(a^2 - c^2)$.

8. $4a^2x(a + x)$; $21(x + y)(a + b)$; $a(p + q)(p - q)$.

9. $a(a+b)(b+c)$; $x(x+1)$; $x^2(x-3)$; $(x-1)(x+1)$; $a^3(a-b)^3$.

10. $ab(x + a)(x + b)$; $ab(x^2 - a^2)$; $(x - 1)(x + 1)$.

11. $(x - 2)(x - 1)$; $21(x - 2)(x+2)$; $x(x+1)(x+2)$; $(x+y)^3$.

12. $(x + 1)(x + 2)(x + 3)(x + 4)$; $(a + b)(a - b)$.

13. $x(3x - 2)(2x - 5)(x + 7)$; $x^2y(a^2 - b^2)$.

14. $x^3(x^2 - a^2)$; $x^3(x - a)(x - b)(x - c)$.

15. $6(x - y)^3$; $6(x + y)^5$; $(x + b)(x^2 - a^2)$.

16. $(2x - 5)(9x^2 - 1)$; $a^4(a + 3)^2$; $(a - b)^2(a + b)^3$.

17. $(a + b)^2(a - x)^2$; $(a + b)^2(a^3 + b^3)$.

18. $(a - x)^2(b - y)^3$; $6(a + 1)^2(a - 1)^2$.

19. $-(1 - 2x)(1 + 2x)$; $(x^2 - y^2)^2$; $a^nb^n(a - 1)(b + 1)$.

20. $-6a^nb^n(b^2 - 1)$; $x^2 - y^2$; $(x^2 - 1)(x^2 + 1)$ or $x^4 - 1$.

[*b*.] (Page 144.)

21. $ab(4a^2 - 1)$; $6x(3x - 1)$; $x(x - 2)(x + 2)$.

22. $(x + 1)(x - 1)^2$; $(x - 1)(x - 2)(x - 4)$;
    $(x - 1)(x + 2)(x - 3)$.

23. $(x + 3)(x + 4)(x + 5)$; $(x - 1)^2(x + 2)$.

24. $(a - 1)(a - 2)(a + 2)$; $(x^2 - y^2)^2$ or $(x - y)^2(x + y)^2$.

25. $(x + 2) (x - 4) (x - 10) (x + 12)$; $(x + 3) (x - 3) (x - 12)$.

26. $(x - 2) (x - 4) (x - 7)$; $(x + 1) (x + 3) (x - 4)$.

27. $(x + a) (a^2 - b^2)$; $(1 - x) (1 + x)^2$.

28. $(x - a) (x + a) (x - b)$; $(a + b - c) (a + b + c)^2$.

29. $(x - 2)^2 (x + 2)^2$; $(x + 3) (x - 3) (2x - 1)$.

30. $(y + 2) (y - 3) (3y + 1)$; $(2x + 3) (2x - 3) (3x - 2)$.

31. $(2x + 3y) (2x - 3y) (3x - 2y)$; $3 (x - 1) (3x - 1) (3x + 2)$.

32. $20a^2y (4a - 1) (5a + 1) (3a + 1)$; $(4a - 1) (4a + 1) (5a + 1)$.

33. $(x + 2) (x - 2) (3x - 7)$; $x^{4n} - y^{2n}$.

34. $(x - 2) (x - 3) (x - 4) (x - 5)$; $2ay^2 (4x - 1) (x + 3) (3x - 2)$.

35. $(x + y) (x - y) (x^2 + y^2)$; $12 (x^2 - 1)^2$.

36. $(x - a)^2 (x + a)^2$ or $(x^2 - a^2)^2$; $-12x (x - 1) (x + 1)^3$.

37. $bc (a^2 - b^2)$; $x^2 (x - 1)^2 (x + 1)^2$.

38. $(x + y) (x^2 + xy + y^2) (x - y)$ or $(x+y) (x^3-y^3)$; $24 (1-x^4)$.

39. $a^4x^4 - b^4y^4$; $a^n - b^n$.

40. $12a^3 (a^2 - y^2)$, $(x^3 + a^3)$.

41. $1 - x^3$; $(1 - 2x)^2 (1 + 2x + 4x^2)$; $(x + y)^3 (x^3 + y^2)$.

42. $(x + 1) (x^2 - x + 1) (x^2 + x + 1)$ or $(x + 1) (x^4 + x^2 + 1)$;
     $(x - y) (x^4 + x^2y^2 + y^4)$.

43. $(x^2 - y^2) (x^4 + x^2y^2 + y^4)$ or $x^6 - y^6$; $(x + 4)^3 (x^2 - 4x + 16)$;
     $(x^n - 1) (x^m + 1)$.

44. $1 + x^2 + x^4$; $(x^2 - y^2) (x^4 + x^2y^2 + y^4)$ or $(x^3 - y^3) (x^3 + y^3)$
     or $x^6 - y^6$.

45. $- (a - b) (b - c) (c - a)$; $- (y^2 - z^2) (x^2 - y^2) (x^2 - z^2)$.

EXERCISE LV.   (Page 145.)

1. $(x + 1) (x + 2) (x + 3) (x + 5) (x - 5) (x - 6)$.

2. $(x + 1) (x^2 + 1) (6x^3 + 5x^2 + 2x - 1)$.

3. $(x + 2) (x + 3) (x + 4) (x + 5)$.

4. $(x + 3) (x + 4)^2 (x + 5)$.

5. $3 (2x + 3) (2x + 5) (x^2 - x - 4)$.

6. $x (x - 1) (x - 4) (1 - 2x) (1 + 2x)$.

7. $6 (x - 1)^2 (x - 2) (x + 2) (x - 4)$.

8. $(x + 4) (x^2 - 2) (4x^2 + 2x + 5)$.

9. $(x + 5) (x^2 - x + 1) (3x^2 + 7x + 6)$.

10. $a(a + 5)(7a + 9b)(4a^2 + 3a + 9)$.

11. $(x - 1)(2x + 3)(3x - 2)(x^2 - x + 1)$.

12. $(x - 1)(x - 3)(x + 3)(x^2 - 3)(x^2 - 2x + 3)$.

13. $(3x^2 + 2x + 1)(3x^3 - 2x^2 + 2x - 1)(2x^3 + 3x^2 - 2x + 1)$.

14. $(x + 1)(x + 2)(x + 3)(x + 4)$.

15. $(x + 1)(x - 3)(3x^2 - 4x + 6)(3x^2 - 6x + 4)$.

EXERCISE LVI [a]. (Page 146.)

1. -30.　　　2. 20.　　　3. $a = b = 12$.　　　4. 10.

5. $a = -10$ and $b = -1$.　　　6. -114.　　　7. 13.

8. $c = 22$, $a = 48$.　　9. $1-a$.　　10. $b = 2$.　　11. $c = b^2(1-a)$.

13. Divisor $= (x - ay)(x - bz)$, and dividend vanishes for each of these factors; *i.e.*, for $x = ay$, $x = bz$, substitute and subtract.

15. Substitute $x = -a$ in each expression; subtract and $a = p-1$; substitute this in $a^2 - qa + 1 = 0$.

[b.]

1. Remainders on dividing by $x + c$ is zero. First quotient $= (x + a - c)$, which multiplied into $x^2 + a'x + b'$, gives required expression.

6. Unity.　　　7. $3x^2 + 2x + 1$.

8. $2x(4x^2 + 1)(5x^2 - 1)^2(5x^2 + x + 1)$.

EXERCISE LVII [a]. (Page 153.)

1. $\dfrac{a + b}{a}$; $\dfrac{b - a}{b}$; $\dfrac{x + 1}{x}$; $\dfrac{x - 1}{x}$; $\dfrac{3x + 2}{x}$; $\dfrac{xy + 1}{x}$; $\dfrac{x^2y - 1}{x}$.

2. $\dfrac{x^2 + x + 1}{x}$; $\dfrac{a^2 - a - 1}{a}$; $\dfrac{a^4 + a^3 + 1}{a^2}$; $\dfrac{6x^3 + 4x^2 - 3}{2x}$.

3. $\dfrac{2x + 1}{x}$; $\dfrac{2x^2 + x - 2}{x}$; $\dfrac{3x^3 - x^2 + 2}{x}$; $\dfrac{3a^2b^2 - 1}{ab}$.

4. $\dfrac{x}{x - 1}$; $\dfrac{3x - 5}{x - 1}$; $\dfrac{x^2(4x^3 + 3)}{x^3 + 1}$; $\dfrac{x(x^3 + 1)}{x^2 + 1}$.

5. $\dfrac{(a + b + c)(a + b - c)}{2ab}$; $\dfrac{(b + c - a)(a - b + c)}{2ab}$; $\dfrac{x^3 + 1}{x - 1}$.

6. $\dfrac{x(x + 3)(x + 5)}{x + 2}$; $\dfrac{x^2 - a^2 + xy + ay + 1}{x + a}$; $\dfrac{3x^2 + 2x + 1}{x + 4}$.

7. $\dfrac{22a^2 - 40ab + 16b^2}{5a - 6b}$; $\quad \dfrac{a^3 + x^3 + a^2 - x^2}{a + x}$.

8. $\dfrac{2x^4 + x^2y^2 + y^4}{x^2 - xy + y^2}$; $\quad \dfrac{x^2 + xy + y^2}{x + a}$; $\quad \dfrac{x^3}{x - 1}$.

9. $\dfrac{3x^4 - 24x^2 + 3a^2}{a^2 + 2ax + x^4}$; $\quad \dfrac{(a - 2b)(a + 2b)(a^2 + 4b^2)}{(a - 3b)(a + 3b)}$; $\quad \dfrac{8x^3 - 27}{2x - 7}$.

10. $\dfrac{a^3 - b^3}{a + b}$; $\quad \dfrac{x^3 + y^3}{x - y}$; $\quad \dfrac{x^3 - y^3}{x + y}$;

$\dfrac{2x(1 - 3x^2 + 3x - 9x^4 + 9x^5 + 27x^6 - 27x^7)}{1 + 3x}$.

[b.]

1. $a + \dfrac{1}{a}$; $\quad a - \dfrac{1}{a}$; $\quad x + \dfrac{y}{x}$; $\quad 1 + \dfrac{b^2}{a^2}$; $\quad a + \dfrac{a^2}{b^2}$; $\quad 2 + \dfrac{a}{x + a}$.

2. $1 + \dfrac{3}{x - 2}$; $\quad 1 + \dfrac{7}{x - 4}$; $\quad 1 + \dfrac{2}{a - 5}$; $\quad 1 + \dfrac{y^2}{x^2 - 2y^2}$;

$1 - \dfrac{2}{a^2 + 1}$; $\quad a - 1 - \dfrac{1}{a + 1}$.

3. $x^2 + x + 1 + \dfrac{2}{x - 1}$; $\quad x + a - \dfrac{a^2}{x - a}$; $\quad x - a + \dfrac{a^2}{x - a}$;

$1 + x + \dfrac{x^2}{1 + x}$.

4. $x - \dfrac{4a^2}{x + 2a}$; $\quad 1 + \dfrac{2}{x^2 - 3x - 1}$; $\quad 1 + \dfrac{a}{2x^2 - 3x + 1}$;

$x^2 - ax + a^2 - \dfrac{5a^3}{x + a}$.

5. $1 - \dfrac{2}{x + 1} + 1 + \dfrac{2}{x - 1}$; $\quad 1 + \dfrac{1}{x - 4} + 1 + \dfrac{1}{x - 5}$;

$1 - \dfrac{4x^2 + 3x - 7}{x^3 + 4x^2 - 5}$.

6. $1 + \dfrac{a + b - c - d}{mx + c + d}$; $\quad 1 + \dfrac{b - c}{mx - b - d}$; $\quad 1 + \dfrac{2}{ax + m - 1}$;

$3x + 1 - \dfrac{11}{6x - 9}$.

7. $1 + \dfrac{2}{ax + m - 2} + 1 + \dfrac{2}{ax + n - 2}$;

$x + 2 + \dfrac{1}{x - 8} + x + 2 + \dfrac{1}{x - 6}$.

8. $12x - 25 + \dfrac{245x - 49}{5x^2 + 9x - 2}$; $x^2 - xy + y^2 - \dfrac{2y^3}{x + y}$;

$a - 6b - c - 1 + \dfrac{16b^2 + 8bc + b}{a + 2b + c}$.

EXERCISE LVIII [a]. (PAGE 156.)

1. $\dfrac{a}{b}$; $\dfrac{c}{d}$; $\dfrac{a^2}{b^2}$; $\dfrac{1}{c}$; $\dfrac{x}{y}$; $\dfrac{ab}{c}$; $\dfrac{x^2}{a^2}$.

2. $\dfrac{ax}{b}$; $\dfrac{x}{4y^2}$; $\dfrac{4b}{5a}$; $\dfrac{2x}{3a^2y}$; $\dfrac{1}{3abc}$.

3. $\dfrac{1}{3bx}$; $\dfrac{x^2}{a^{n-2}}$; $\dfrac{y^n}{3x}$; $\dfrac{a^{m-2}}{b^{m-2}}$; $\dfrac{a-b}{ab}$.

4. $\dfrac{a}{y-x}$; $\dfrac{b}{2a+3c}$; $\dfrac{x+y}{x-y}$; $\dfrac{2a-3b}{3a-2b}$; $\dfrac{a}{a-b}$.

5. $\dfrac{x}{x^2-1}$; $\dfrac{1}{y(a+x)}$; $\dfrac{1}{y}$; $\dfrac{1}{a^2-b^2}$; $\dfrac{a+b}{a-b}$.

6. $\dfrac{a+b}{a^2+ab+b^2}$; $\dfrac{x^2-x+1}{x^2+2x+1}$; $\dfrac{a^2+ab+b^2}{a+b}$; $\dfrac{x}{x-8}$; $\dfrac{x-3}{x-5}$.

7. $\dfrac{x+4}{x+7}$; $\dfrac{x+7}{x-3}$; $\dfrac{x+7a}{x-3a}$; $\dfrac{x+4}{x+3}$.

8. $\dfrac{x^2+x+1}{x^2-2x+1}$; $\dfrac{x+b}{x+c}$; $\dfrac{x+b}{x-c}$.

9. $\dfrac{3(a+7b)}{4(a+5b)}$; $\dfrac{a+b-c}{a+b+c}$; $\dfrac{a^2-b}{b^2-a}$; 1.

10. $\dfrac{2x+a+b}{2x-a-b}$; y in numerator should be b and in denominator x;

$\dfrac{1}{a-b}$; $\dfrac{a+b}{a-2b}$; $\dfrac{5a^2+3b}{a^2-b}$.

11. Irreducible; $\dfrac{y+z}{z-y}$; $\dfrac{3x+2}{4x+5}$; $\dfrac{(a+b)^2}{a-b}$.

12. $\dfrac{x^2-xy+y^2}{x^2-2xy+y^2}$; $\dfrac{a+b}{a-b}$; $\dfrac{x^2+1}{x^2-1}$.

[b.]

1. $\dfrac{x-11}{4x^2-5x-5}$; irreducible; $\dfrac{x+4}{(x-1)^2}$.

2. $\dfrac{7x - 2y}{5x^2 - 3xy + 2y^2}$; $\dfrac{x^2 + x - 2}{x^2 + 5x + 5}$; $\dfrac{5a^3(a + x)}{x(a^2 + ax + x^2)}$;

in first term of numerator, x should be a.

3. $\dfrac{9(x^2 + y^2 + z^2 - xy - xz - yz)}{2x - y - z}$; $\dfrac{ax + by}{ax - by}$.

4. $\dfrac{x + 2}{2x - 1}$; $\dfrac{(x + 1)(3x - 7)}{(x - 1)(7x + 3)}$.

5. $\dfrac{a - b}{1}$; $\dfrac{(2x - 3a)^2}{1}$; $\dfrac{x^3 - 2x^2 + 12x - 18}{x^3 - 2x^2 + x + 4}$.

6. $\dfrac{3(x - 3a)(x - 4a)}{2(x + 3a)(x + 4a)}$; $\dfrac{a(x + 8a)}{x(x + 7a)}$.

7. $\dfrac{3ax^2 + 1}{4a^2x^4 + 2ax^2 - 1}$; $\dfrac{c}{1}$.

8. $\dfrac{a^2 + b^2}{a}$; $\dfrac{x - 5}{x + 5}$; irreducible.

9. $\dfrac{a^2 + b^2 + c^2 - ab - bc - ca}{a^2 + b^2 + c^2 + 2ab + 2bc + 2ca}$; $\dfrac{x + y + z}{2}$.

10. $\dfrac{1}{3(a + b + c)}$; Expre'n $= \dfrac{(x - 1)(x^{n-1} + x^{n-2} + \ldots + x - n)}{(x - 1)\{nx^n - (x^{n-1} + x^{n-2} + \ldots + 1)\}}$

$= \dfrac{(x - 1)(x^{n-1} - 1 + x^{n-2} - 1 + \ldots + x - 1)}{(x - 1)(x^n - x^{n-1} + x^n - x^{n-2} + \ldots x^n - 1)} =$

$\dfrac{(x - 1)^2\{x^{n-2} + 2x^{n-3} + 3x^{n-4} + \ldots + (n - 1)\}}{(x - 1)^2\{x^{n-1} + x^{n-2}(x + 1) + x^{n-3}(x^2 + x + 1) + \ldots + 1\}}$

$= \dfrac{x^{n-2} + 2x^{n-3} + 3x^{n-4} + \ldots + (n - 1)}{x^{n-1} + x^{n-2}(x + 1) + x^{n-3}(x^2 + x + 1) + \ldots + 1}$

$= \dfrac{\text{last numerator}}{nx^{n-1} + (n-1)x^{n-2} + (n-2)x^{n-3} + \ldots + 1}$.

11. $\dfrac{(a + b)(b + c)(c + a)}{1}$; $\dfrac{x + 2y + 3z}{x - 3y - 4z}$.

12. $\dfrac{5xy(x + y)}{2(x^2 + xy + y^2)}$; $\dfrac{a^2 + b^2 + c^2 + ab + bc + ca}{5(a^2 + b^2 + c^2 - ab - bc - ca)}$.

EXERCISE LIX [a]. (Page 162.)

1. $\dfrac{3}{x}$; $\dfrac{1}{x}$; $\dfrac{a}{x}$; $\dfrac{x + y}{a + b}$; $\dfrac{12}{y}$.

2. $\dfrac{4(a + b)}{5}$; $\dfrac{2(a - b)}{7}$; $-\dfrac{2b}{b} = -2$.

3. $\dfrac{a+b}{ab}$; $\dfrac{b-a}{ab}$; $\dfrac{a^2-x^2}{ax}$; $\dfrac{a+b}{abx}$; $\dfrac{a(a+x)}{x^2}$; $\dfrac{a(a-1)}{x}$.

4. $\dfrac{5x-17}{6}$; $\dfrac{x+17}{6}$; $\dfrac{12a-5}{35}$; $\dfrac{2a^2b-1}{10b^2}$.

5. $\dfrac{7x+2}{12}$; $\dfrac{x(b-a)}{ab}$; $\dfrac{ab+bc+ca}{abc}$; $\dfrac{a^2b^2-c^2}{abc}$.

6. $\dfrac{a+b+c}{abc}$; $\dfrac{x^2+y^2+z^2}{xyz}$; $\dfrac{bcx+acy+abz}{abc}$; $\dfrac{x^2+18x-27}{9x^2}$.

7. $\dfrac{6a+4b+3c}{12x}$; $\dfrac{x(az+ay+yz)}{ayz}$; $\dfrac{a+bx+cx^2}{x^n}$; $\dfrac{x^2-a^2b^2}{abx}$.

8. $\dfrac{2x}{x^2-1}$; $\dfrac{2b}{a^2-b^2}$; $\dfrac{2x^2}{x^2-a^2}$; $\dfrac{a^2+x^2}{a^2-x^2}$.

9. $\dfrac{11}{x^3+3x-28}$; $\dfrac{c-a}{(a+b)(b+c)}$; $\dfrac{x+4}{x^2+5x+6}$; $\dfrac{2a(a-1)}{a^2-b^2}$.

10. $\dfrac{2(x^2+1)}{x^2-1}$; $\dfrac{2(x^2+a^2)}{x^2-a^2}$; $\dfrac{2x^2-32x+127}{x^2-17x+72}$; $\dfrac{24x}{4x^2-9}$.

11. $\dfrac{x^2-2xy+y^2}{4y(x+y)}$; $\dfrac{3(2x+7)}{(x-1)(x-2)(x+4)}$; $\dfrac{1+a}{x+a}$; $\dfrac{2(1+nx)}{(n^2-1)(1-x^2)}$.

12. $\dfrac{2x^3}{x^4+x^2+1}$; $\dfrac{3ax}{x^3+a^3}$; $\dfrac{3ax}{x^3-a^3}$.

13. $\dfrac{2a}{a^2-x^2}$; $\dfrac{2a}{a(a-x)}$; $\dfrac{1}{a}$.

14. 1; $\dfrac{a}{(1-a)^3}$; $\dfrac{2(x^2-ax+a^2)}{x^2-a^2}$.

15. $\dfrac{2x^2-2xy-y^2}{x^2-y^2}$; $\dfrac{4xy}{(x^2-y^2)^2}$; $\dfrac{a+b}{a^2-ab+b^2}$; $\dfrac{a-b}{a^2+ab+b^2}$; $x^{2n}+2$.

16. $\dfrac{2x-7}{(x-2)(x-3)(x-4)}$; $\dfrac{2(a+x)}{a^2+ax+x^2}$.

[b.]

1. $\dfrac{1-6x^2}{1-4x^2}$; $\dfrac{x}{4x^4-y^2}$.

2. $\dfrac{4a^2+b^2}{4a^2-9b^2}$; $\dfrac{4}{a-b}$.

3. $\dfrac{18}{x^3-27}$; $\dfrac{2x^2}{x^3-8}$.

4. $\dfrac{16x^2+29x-104}{12(x-5)(x+5)}$; $\dfrac{x+3}{x^4-1}$.

5. $\dfrac{x + 11}{(x - 1)(x + 4)}$; $\dfrac{2x^3 - 5x^2y + 10xy^2 + 5y^3}{(x^2 - y^2)^2}$.

6. $\dfrac{a + b + c}{(a + b - c)(a + c - b)(b + c - a)}$; $\dfrac{6}{(x - 1)(x + 1)(x + 3)}$.

7. $\dfrac{-1}{x + 3y}$; $\dfrac{2916}{x^6 - 6561}$.   8. $\dfrac{8a^7}{x^8 - a^8}$; $\dfrac{a^2}{(a - b)(a - c)}$.

9. $\dfrac{1}{a + b}$.   10. $\dfrac{1}{x + 2}$; $\dfrac{4(ab - cd)}{a^2 + c^2 - b^2 - d^2 + 2ac - 2bd}$.

11. 1 ; 0.   12. $\dfrac{a}{xyz}$; $\dfrac{x(1 - 3x + 3x^2)}{(1 - x)^3}$.

13. $\dfrac{1 + 2x + 3x^2}{4(1 - x^4)}$; $\dfrac{x + c}{(x - a)(x - b)}$.

14. $\dfrac{4a^m b^m}{a^{2m} - b^{2m}}$; $\dfrac{y^{n-1}z}{(y - z)^n}$.

15. $\dfrac{a^y b + a^z c + a^u d - c}{a^z}$; $\dfrac{8x^2 + 4x - 3}{(x - 1)(x + 1)(2x + 1)}$.

16. $\dfrac{a(a^2 + 2ax + 3x^2)}{4(a^4 - x^4)}$; $\dfrac{2x}{x + y}$.

17. $\dfrac{2 + x^2 - x^3}{2(x^2 + 1)(x^3 + 1)}$; 2.   18. $\dfrac{1}{(x + 1)(x + 2)(x + 3)}$.

[c.]

1. $\dfrac{4a^2 - 2ab}{a^2 - b^2}$, ab for a^2b in num. and denom.; $\dfrac{b^2(a^3 - a^2 + b^3)}{a^6 - b^6}$.

2. $\dfrac{16x^{15}}{1 - x^{16}}$; $\dfrac{q(b - a)}{(x - a)(x - b)}$.

3. $\dfrac{(1 - x)(1 + 2a + a^3)}{(1 - a)(1 + a)(1 + a^2)}$; $\dfrac{p(a - c)(x - b) + q(b - c)(x - a)}{(x - a)(x - b)(x - c)}$.

4. $\dfrac{24(x - 9)}{(x - 1)(x - 3)(x - 5)(x - 7)}$;

    $\dfrac{-22x^3 + 44x^2 - 180x - 622}{7(x - 1)(x + 2)(x - 3)(x + 4)}$.

5. 0.   6. 0.   7. 1.   8. $a + b + c$.

9. 0.   10. 0.   11. 0.   12. 0.   13. 0.   14. 0.   15. 1.

16. $(x^3 - y^3)$; sign $+$ should be $-$ between given quantities.

17. $\dfrac{a^z b \, (a^{2z} + b^2)}{(a^{2z} - a^z b + b^2)(a^{4z} - a^{3z} + a^{2z}b^2 - a^z b^3 + b^4)}$.

18. $\dfrac{a^{4z}\left(a^{2y} - \dfrac{1}{a^{2y}}\right) + a^{2z}\left(a^{2y} + \dfrac{1}{a^{2y}}\right) - (a^{4z} + 1)}{1 - a^{4z}}$.

EXERCISE LX [a]. (PAGE 170.)

4. $c^2 \div b^2$; $a^2 \div c^2$; $b \div a$; $\dfrac{x^4}{a^4}$; $b^2 \div a^2$; $b + x$.

5. $(a + b) \div (a - b)$; $4a^5 \div 15x^6$; $\frac{5}{3}a^2x$; 1.

6. $\dfrac{a^2 - 9}{a^2 - 16}$; 1; $\dfrac{x - y}{x^2 + y^2}$; $(x + 3)(x + 1)$; $1 \div x$.

7. $\dfrac{1}{(x + 4)^2}$; 1; $-\dfrac{a}{b}$; $\dfrac{1}{z} - \dfrac{1}{x}$; $\dfrac{5b}{3a^{2m}c}$.

8. $xy + yz + zx$; $2(a + b + c)$; $ab + bc + ca$.

9. $2 + 2x^2$; $4ab$; $\dfrac{1}{a(x - a)}$; $\dfrac{n}{m}$

10. $\dfrac{1}{a^2} + \dfrac{1}{b^2} + \dfrac{1}{c^2}$; $\dfrac{2(a^2 + b^2)}{(a^2 - b^2)^2}$; 1; $\dfrac{1 - a}{1 + b}$.

11. $- 2x$; $\dfrac{x^2}{y^2}$; $\dfrac{a^{3x-1}b^{4y-1}}{c^{z+1}}$.

12. $\dfrac{ax}{(x - b)^2}$; $\dfrac{x}{x - 3}$; $x - 1$; $(x - a)^5 \div a^2$.

13. $\left(\dfrac{x + 3}{x - 3}\right)^3$; $\dfrac{a}{b}$; $\dfrac{a(a^2 + ax + x^2)}{(a - x)(a + x)^2}$.

14. $\dfrac{x^2 + xy + y^2}{x + y}$; $\dfrac{1}{x^3 + a^3}$; $\dfrac{a^2 - ab + b^2}{(a + b)(a - b)^2}$; $\dfrac{1 - b^n + b^{2n}}{(1 + b^n)(1 - b^n)^2}$.

15. $\dfrac{(x^2 - 1)(x + 1)}{x^2 + x + 1}$; 1; $\dfrac{(x + b)(x - c)}{(x + a)(x + c)}$.

16. $\dfrac{x - 3b}{x - 2c}$; $\dfrac{(x - c)^2}{(x + a)(x - b)(x + b)}$.

17. $\dfrac{x + b}{x + c}$; $(x + 2)$; $\dfrac{1}{x^2} - 1$.

[b.]

1. $\dfrac{1}{(x-y)^2}$; $\left(\dfrac{5ab^2}{2}\right)^m$.   2. 1 ; $\dfrac{(x^2-1)(x+1)}{(x-7)(x-5)^2}$.

3. $\dfrac{x+a}{x-a}$; $\dfrac{a}{b}$.   4. $\dfrac{a-b}{a+b}$; $\dfrac{2ab^6}{15cd^6}$.

5. $\dfrac{x^2(x-1)}{x^2-4)}$; $(100xy)^m$; $\dfrac{a^2}{b^2}+\dfrac{b^2}{a^2}-1$.

6. $\dfrac{a^2(x+13a)}{b^2c(x-2a)}$, put x for a in first term of numerator and a

   for a^2 in second term ; $\dfrac{625a^5}{864b^2}$.

7. $\dfrac{a+b-c}{b-c-a}$; 1.   8. $\dfrac{x^4}{x^2+y^2}$; $\left(\dfrac{5bc^3d^2e}{3a^2f^5}\right)^m$.

9. $\dfrac{(x+a)(x^3+a^3)}{(x^2+a^2)^2(x^2+ax+a^2)}$; $\dfrac{1}{(x-4)(3x-2)}$.

10. $\dfrac{1-y}{x}$; 1.   11. $\dfrac{x^2-6x-8}{x-12}$; 1.

12. $\left(\dfrac{x+y}{x-y}\right)^3$; $\dfrac{c-d}{(a+b)^4}$.   13. $\dfrac{(x-a)^2(x+3a)}{(x-c)^2(x+6a)}$; $\dfrac{(5a^{2m})^z}{7^{mz-z}}$.

14. $abcde$; $\dfrac{y^6}{(y-x)(x^2+y^2)}$.   15. 2 ; 0.

16. 1 ; $\dfrac{675cd}{8ab}$.   17. $\dfrac{abc}{(a-c+b)(ab+bc+ca)}$.

18. $\dfrac{ax^3(x^2+a^2)}{x^3+a^3}$; $-\dfrac{p^4q^4}{p^4+p^2q^2+q^4}$.

EXERCISE LXI [a]. (PAGE 176.)

1. $\dfrac{4a-3}{16}$; $\dfrac{8a+1}{28}$; $\dfrac{x-3}{6}$; $\dfrac{a^2-bc}{b^2}$; $\dfrac{a^2}{b^4}$; ab.

2. $\dfrac{2(a-b)}{3}$; $\dfrac{16+x}{16-x}$; $\dfrac{6x-20}{12x+5}$; $\dfrac{2x-5}{x-10}$; $\dfrac{12x-8}{12x+9}$.

3. $-\dfrac{1}{b}$; $\dfrac{a+b}{a-b}$; 1 ; $\dfrac{b+a}{b-a}$; $\dfrac{x}{2+x}$.

4. $\dfrac{a^2}{b^2}$; $\dfrac{24b+9a}{12a+8b}$; $\dfrac{x^2-1}{x^2+1}$; 1

5. $\dfrac{a - ax - 1}{a - ax + 1}$; $\dfrac{1}{a}$; $\dfrac{2ax}{a^2 + x^2}$; $\dfrac{2x - 35}{x^2}$.

6. $\dfrac{x^3 - 2x^2 - 3}{x^3(3 - x)(3 + x)}$; $\dfrac{1}{x}$; $a + x$.

7. $\dfrac{a^3 + x^3 - a^2}{a^3 + x^3 - x^2}$; $\dfrac{12}{(x - 1)(x - 2)}$; $-\dfrac{a}{x}$.

8. $\dfrac{(a - b)^2}{a^2}$; $(a + 1)^2$; $a^2 - ax + x^2$.

9. $\dfrac{2ab}{a^2 + b^2}$; $\dfrac{4ab(a^2 + b^2)}{a^4 + 6a^2b^2 + b^4}$; $\dfrac{(a^2 + b^2)(a^4 + 14a^2b^2 + b^4)}{2ab(3a^4 + 20a^2b^2 + 3b^4)}$.

10. $\dfrac{b - a + 4}{b - a + 5}$; $\dfrac{6a}{1 - 2x}$.

11. $\dfrac{a^2 + b^2}{b - a}$; $\dfrac{ab + bc + ca}{a^2c + ab^2 + bc^2}$.

[b.]

1. $\dfrac{4ab(a + b)}{(a - b)(a^3 + 2a^2b + ab^2 - a^2 - b^2)}$; $\dfrac{4(ax^2 - 4)}{x^2(3x - 2)(3x + 2)}$;

$\dfrac{1 + x}{1 + x^2}$.

2. $\dfrac{1}{(x^2 - y^2 - z^2)(x^2 + y^2 - z^2)}$; $\dfrac{2}{63}$; $\dfrac{a - 3}{(a - 1)(a + 3)}$.

3. $\dfrac{b^2 - ab}{a^2 + ab}$; x^2y^2.

4. $\dfrac{1}{x^3 + 1}$; $\dfrac{x(x^2 + 2)}{x^4 + 3x^2 + 1}$.

5. 1 ; $\dfrac{1}{a^2b(b - a)}$.

6. 0 ; $\dfrac{x + 2y + 6}{8x(y + 6)}$.

7. $-x$; $\dfrac{1 + 2x}{2 + 3x}$.

8. $\dfrac{a^6}{(a + x)(a^2 - x^2)}$; $\dfrac{1}{x^3}$.

9. $\dfrac{4a^2b^2}{(a^2 - b^2)^2}$; 1.

[c.]

1. 1 ; $a + b$.
2. $16x + 11y$.
3. $5x + 11y$.

4. $\dfrac{x - y}{x + y}$; $\dfrac{3 - 3x}{4 + 4x}$.
5. 1.
6. $\dfrac{2}{x}$.

7. $\dfrac{(a-b)}{(a+b)}$.   8. $\dfrac{a}{2x^2}$.   9. $\dfrac{ab}{a+b}$.

10. 2.   11. $-\frac{1}{22}$ or $-\frac{9}{22}$.   12. c.

16. Take the fractions in pairs, thus :
$$\left(\frac{1}{s-a}+\frac{1}{s-b}\right)+\left(\frac{1}{s-c}-\frac{1}{s}\right)=\frac{c}{(s-a)(s-b)}+\frac{c}{s(s-c)},$$
by substituting for $2s$, etc.

17. a.   20. Multiply given relations out and transpose,
$$a+b+c+d=abc+abd+bcd+acd=abcd\left(\frac{1}{a}+\text{etc.}\right)\text{etc.}$$

EXERCISE LXII. (Page 185.)

21. 1. $1+x+x^2+x^3+x^4+\ldots$

2. $1+3x+9x^2+27x^3+81x^4+\ldots$

3. $1-x+x^2-x^3+x^4-\ldots$

4. $1-3x+9x^2-27x^3+81x^4-\ldots$

5. $\dfrac{a^2}{x}+\dfrac{a^2b}{x^2}+\dfrac{a^2b^2}{x^3}+\dfrac{a^2b^3}{x^4}+\dfrac{a^2b^4}{x^5}+\ldots$

6. $x+\dfrac{x^2}{a}+\dfrac{x^3}{a^2}+\dfrac{x^4}{a^3}+\dfrac{x^5}{a^4}+\ldots$

7. $a+abx+ab^2x^2+ab^3x^3+ab^4x^4+\ldots$

8. $1-\dfrac{2x}{a}+\dfrac{3x^2}{a^2}-\dfrac{4x^3}{a^3}+\dfrac{5x^4}{a^4}-\ldots$

9. $1+x-x^3-x^4+x^6+\ldots$

10. $1+ax+a^2x^2+a^3x^3+a^4x^4+\ldots$

27. $\dfrac{x^3+y^3}{x^3-y^3}=\dfrac{(x+y)^3-3xy(x+y)}{(x-y)^3+3xy(x-y)}=\text{etc.}$
Substitute from given conditions.

EXERCISE LXIII [a]. (Page 191.)

1. $x=7$.   2. $x=1\frac{1}{2}$.   3. $x=2$.

4. $x=3$.   5. $x=4$.   6. $x=\frac{21}{11}$, read x for $2x$.

7. $x=\frac{1}{2}$.   8. $x=5$.   9. $x=6$.

10. $x=\dfrac{a(1+b)}{a-2}$.   11. $x=\dfrac{3a-b}{2}$.   12. $x=1$.

13. $x = \dfrac{a^2(b-a)}{b(a+b)}$.   14. $x = \dfrac{3a-6}{4}$.   15. $x = \dfrac{2ab}{a+b}$.

16. $x = \dfrac{d(a+c)}{b}$.   17. $x = b$.   18. $x = 9$.

19. $x = -\frac{3}{4}$.   20. $x = 3$.   21. $x = 60$.

22. $x = 1$.   23. $x = -\frac{1}{12}$.   24. $x = -\frac{2}{7}$.

25. $x = \dfrac{ab}{a+b-c}$.   26. $x = \dfrac{a^2}{b-a}$.   27. $x = -6\frac{5}{6}$.

28. $x = -\frac{7}{18}$.   29. $x = -\frac{3}{2}$.   30. $x = 3a$.

31. $x = \dfrac{bn+dm+amn+cmn}{b+d+am+cn}$.   32. $x = 14$.

33. $x = \dfrac{acn-abn-abm-bcm}{nb-nc-ma-mc}$.   34. $x = 3a$; right mem. sh. be $2\frac{1}{6}$.

35. $x = 7$.   36. $x = 3$.   37. $x = 3$.

38. $x = \frac{1}{2}$.   39. $x = 11$.   40. $x = -6$.

[b.]

1. $x = -3\frac{1}{2}$.   2. $x = -107$.   3. $x = \dfrac{a+b+c+d}{n-m}$.

4. $x = 3$.   5. $x = \dfrac{b(a-b+c)}{a}$.   6. $x = \dfrac{1}{ab}$.

7. $x = 0$ or $-4\frac{1}{3}$.   8. $x = -\frac{7}{16}$.   9. $x = \dfrac{m}{c}$.

10. $x = 15$.   11. $x = 3$.   12. $x = 1\frac{1}{2}$.

13. $x = -2\frac{1}{2}$.   14. $x = -6$.   15. $x = 7$.

16. $x = \dfrac{m}{2}$.   17. $x = \pm 3$ or ∞.

18. $x - 1 = 0$ and $4x^2 + 5x + 3 = 0$.   19. $x = a + b$.

20. $x = 2$.   21. $x = a$.   22. $x = \dfrac{7bc}{9b+4c}$.

[c.]

1. $x = 13$; second numerator should be 3.   2. $x = -9$.

3. $(x-3)(2x-5) = 0$; $x = 3$.

4. $x = 4\frac{1}{2}$.   5. $x = -1$.

6. $x = \dfrac{ab - bc - ca}{c^2}.$ 7. $x = \dfrac{a(a - b)(a - c)}{(a + b)(a + c)}.$

8. $x = 0.$ 9. $x = 5.$

10. $x(b - a) = 0$; whence $x = 0$, unless $b - a = 0$, in that case x may have any finite value.

11. $x = \pm \sqrt{5}$ or $0.$ 12. $x = c.$

13. $x(2x + 5) = 0$; $\therefore x = 0$ or $-2\frac{1}{2}.$

14. $x = a + b + c.$ 15. $x = a^2 + b^2 + c^2.$

16. $x = a^4.$ (First numerator on right hand should be $x - 1.$)

17. Take in pairs the fractions with like numerators;
$$x = \frac{np(c - a) + mp(a - b) + mn(b - c)}{m(a - c) + n(b - a) + p(c - b)}.$$

18. $(-7x + 49)\left\{\dfrac{1}{x^2 + x - 2} - \dfrac{1}{x^2 + x - 12}\right\} = 0;$
$\therefore x = 7$ or $\infty.$

19. Complete the divisions, cancel and transpose;
$$\therefore \frac{2}{x - 4} - \frac{1}{x - 5} - \frac{1}{x - 2} = 0;$$
or $\dfrac{1}{x - 4} - \dfrac{1}{x - 2} = \dfrac{1}{x - 5} - \dfrac{1}{x - 4};$
whence $(x - 8)(x - 4) = 0$; $\therefore x = 8.$
The value 4 is not admissible.

20. $x = \dfrac{bn(q - p)(m - p) + ap(q - n)(m - n)}{b(q - p)(m - p) + a(q - n)(m - n)}.$

21. $x = \{m(b - c) - n(a + c)\} \div (m - n)$

22. $(a^2x + b^2x - a^2b - ab^2 - b^2c + a^2c) \times$
$$\left\{\frac{1}{(x - a)(x - b)} - \frac{1}{(x - a - c)(x - b + c)}\right\} = 0.$$
$\therefore x = \{a^2(b - c) + b^2(c + a)\} \div (a^2 + b^2).$

23. $(a - b)\left(\dfrac{x}{n - o} - \dfrac{1}{p - q}\right) = 0,\quad x = \dfrac{n - o}{p - q}.$

24. $\dfrac{x - 2a}{b + c - a} - 1 + $ anal. $+$ anal. $= 0$, whence
$$(x - a - b - c)\left\{\frac{1}{b + c - a} + \text{anal.} + \text{anal.}\right\} = 0;$$
$x = a + b + c.$

25. $\dfrac{a-x}{a^2-bc} - \dfrac{1}{a+b+c} + \&c. + \&c. = 0,$ or

$\dfrac{ab+bc+ca-(a+b+c)x}{(a^2-bc)(a+b+c)} + \text{anal.} + \text{anal.} = 0;$

$x = (ab+bc+ca) \div (a+b+c).$

EXERCISE LXIV [a]. (PAGE 198.)
PROBLEMS.

1. $10\frac{5}{12}$ dozen.　　2. $36000.　　3. 12 years.

4. $300.　　5. $\dfrac{ma-12b}{12-m}$.　　6. $\dfrac{abc}{b+c}$.

7. 754.　　8. $3.75.　　9. 142857.

10. $8000.　　11. $190\frac{59}{121}$ bushels.　　12. 90 and 91.

13. Equation reduces to $(4-4)x+40=0$; $x=\infty$; i. e., conditions of problems are inconsistent. In fact, area will always be 45 ft. less, under the given conditions; for using 45 for 85, the resulting equation is an identity.

14. 1857.35\frac{343}{673}$ and 142.64\frac{328}{673}$.

15. 857142; x representing number, equation is
$\frac{1}{10}(x-2)+200,000=\frac{1}{3}x.$　　　　16. $7.60.

17. 550; read 6 in first line; 4 times and 6 cts. in second line.

18. $13\frac{3}{4}$ feet and $16\frac{1}{2}$ feet.

19. A, $2800; B, $3900; C, $5138; D, $2196; E, $2966.

20. $14.

[b.]

1. 960 gallons.　　2. 420 acres.　　3. $1280, $7\frac{1}{2}$%.

4. Gain or loss $\% = \dfrac{q(100+n) \sim 100p}{p}$, according as

$q \begin{smallmatrix}>\\<\end{smallmatrix} \dfrac{100p}{100+n}.$

5. B makes 1740 yds. in 4 m. 34 sec.; C makes 1700 yds. in 4 m. 32 sec. Let $x=$ time in min. from starting at which A overtakes B, then $\dfrac{x}{4\frac{1}{2}}\cdot 1760 = 20 + \dfrac{x}{4\frac{17}{30}}\cdot 1740$, $x=1\frac{611}{622}$ min., distance $775\frac{95}{311}$ yds. from start. Similarly A is found to pass C in $3\frac{2}{8}$ m.; distance $1456\frac{16}{9}$ yds. from start.

6. $10\frac{95}{121}$ miles. 7. 5 gal. 8. $7400. 9. 57 miles.

10. $mnpqr \div (mnpq - mpq - npq - mnq - mnp)$.

11. 484. 12. 1, 2, and 3.

13. 12000 sq. yds.; 45 cts. 14. 189.

15. $x =$ distance; then $\dfrac{2x}{b} = \dfrac{x}{ap} + \dfrac{px - x}{bp}$.

16. Let $2x$ and x be digits; $(2001x)^2 - (1002x)^2 = 2999997x^2 = 4x^2 \times 749,999\frac{1}{4}$. 17. 180,000.

18. $\left(\dfrac{n+1}{n}\right)^{n+1}$. 19. $\dfrac{p\,(11m - 21n)}{20\,(m - n)}$.

20. Regular rate 40 miles, diminished rate $38\frac{2}{21}$ miles; 100 miles.

21. $221 : 273 :: 187 : 231$. 22. $\dfrac{ap - an}{m - n}$. 23. $\frac{1}{2}$. 24. 14172.

EXERCISE LXVII. (Page 212.)

1. $x = 3$; $y = 2$. 2. $x = 6$; $y = -4$.

3. $x = 1$; $y = 3$. 4. $x = 5$; $y = 4$.

5. $x = 4$; $y = -3$. 6. $x = -2$; $y = 3$.

7. $x = 1$; $y = 2$. 8. $x = -20\frac{5}{6}$; $y = -19\frac{1}{3}$.

9. $x = 10$; $y = 9$. 10. $x = \frac{541}{303}$; $y = \frac{1275}{1212}$.

EXERCISE LXVIII. (Page 213.)

1. $x = 2$; $y = 3$. 2. $x = \frac{76}{45}$; $y = -\frac{14}{9}$.

3. $x = 3$; $y = -2$. 4. $x = 5$; $y = -5$.

5. $x = 4$; $y = 4$. 6. $x = \frac{232}{71}$; $y = -\frac{79}{71}$.

7. $x = 3$; $y = 2$. 8. $x = 2$; $y = 1$.

9. $x = 4$; $y = -3$. 10. $x = 3\frac{1}{3}$; $y = \frac{1}{2}$.

EXERCISE LXIX. (Page 214.)

1. $x = 5$; $y = 7$. 2. $x = 1$; $y = -1$.

3. $x = 2$; $y = -3$. 4. $x = -2\frac{21}{34}$; $y = 5\frac{13}{34}$.

5. $x = 5$; $y = -4$. 6. $x = \frac{2}{9}$; $y = \frac{1}{6}$.

7. $x = 1\frac{61}{106}$; $y = 2\frac{38}{159}$. 8. $x = 5$; $y = 6$.

9. $x = 1\frac{398}{559}$; $y = -1\frac{59}{129}$. 10. $x = 3\frac{5}{29}$; $y = -3\frac{16}{29}$.

EXERCISE LXX. (Page 216.)

1. $x = \dfrac{a + b}{2}$; $y = \dfrac{a - b}{2}$. 2. $x = \dfrac{1 - b^2}{a - b}$; $y = \dfrac{ab - 1}{a - b}$.

3. $x = \dfrac{mp - nq}{m^2 - n^2}$; $y = \dfrac{np - mq}{m^2 - n^2}$.

4. $x = 1$; $y = 1$. 5. $x = a + b$; $y = -1$.

6. $x = \dfrac{b(4b^2 - 7a^2)}{4b^2 - 3a^2}$; $y = \dfrac{a(9a^2 - 4b^2)}{3a^2 - 4b^2}$.

7. $x = y = \dfrac{2}{a + b}$. 8. $x = m + n$; $y = m - n$.

9. $x = y = \dfrac{m}{a + c}$.

10. $x = \dfrac{a(bc - 2ac - c^2 - a^2 + 3ab)}{2(2ab + bc - ac - c^2)}$; $y = \dfrac{a(3a^2 + ab + 4ac + ac^2 - bc)}{2(2ab + bc - ac - c^2)}$.

EXERCISE LXXI [a]. (Page 218.)

1. $x = 6$; $y = 12$. 2. $x = 4$; $y = 3$.

3. $x = 7$; $y = 10$. 4. $x = 13\frac{2}{19}$; $y = -4\frac{17}{19}$.

5. $x = 6$; $y = 12$. 6. $x = 4\frac{5}{32}$; $y = -12$.

7. $x = 8$; $y = -\frac{1}{2}$. 8. $x = 2$; $y = 7$.

9. $x = y = 5$. 10. $x = 4$; $y = 5$.

11. $x = 8\frac{1}{2}$; $y = -\frac{1}{2}$. 12. $x = y = \dfrac{a^2b^2}{a^2 + b^2}$.

13. $x = \dfrac{2mn(n^2 - m^2)}{n^4 + 6m^2n^2 + m^4}$; $y = \dfrac{n^4 - m^4}{n^4 + 6m^2n^2 + m^4}$.

14. $x = \dfrac{a^2}{a - b}$; $y = \dfrac{b^2}{b - a}$.

15. $x = 4$; $y = 1$. 16. $x = 6$; $y = -2$.

17. $x = -2$; $y = 4$. 18. $x = 3$; $y = -4$.

[b.]

1. $x = y = \frac{1}{16}$. 2. $x = 14$; $y = -14$.

3. $x = 4\frac{1}{31}$; $y = 11\frac{7}{31}$. 4. $x = 21$; $y = 20$.

5. $x = 3$; $y = 7$.

6. $x = \dfrac{ab - 1}{(1 - a)(1 - b)}$; $y = \dfrac{a - b}{(1 - a)(1 - b)}$.

7. $x = \dfrac{ab + 4b - 2c}{ab - b + 2a}$; $y = \dfrac{a^2 + 2a + ac - ab + b - c}{ab - b + 2a}$.

8. $x = \pm 9$; $y = \pm 3$.   9. $x = a$; $y = b$.

10. $x = 9$; $y = 2$.   11. $x = \dfrac{a}{a - b}$; $y = \dfrac{b}{a + b}$.

12. $x = 8$; $y = 2$.

13. $x = \dfrac{b + c - a - d}{4(bc - ad)}$; $y = \dfrac{c + d - a - b}{2(bc - ad)}$.

14. $x = -2\frac{3}{5}$; $y = -6\frac{1}{5}$.   15. $x = y = 3\frac{1}{2}$.

16. $x = 6\frac{2}{3}$; $y = 8$.

17. $x = \dfrac{b'c - bc'}{ab' - a'b}$; $y = \dfrac{ac' - a'c}{ab' - a'b}$.

   1. $x = y = \infty$, and the equations are *inconsistent*: thus, put $\dfrac{a}{a'} = \dfrac{b}{b'} = k$, and $\therefore a = ka'$, $b = kb'$, and substituting these values of a and b in $ax + by = c$, we get $a'x + b'y = \dfrac{c}{k}$, which is inconsistent with the second given equation.

   2. $x = y = \frac{0}{0}$, *i. e.* the equations are *not independent*: thus, put $\dfrac{a}{a'} = \dfrac{b}{b'} = \dfrac{c}{c'} = m$. Then $a = ma'$, $b = mb'$, $c = mc'$, and substituting in (1), we get $ma'x + mb'y = mc'$, which is a *multiple* of the second given equation.

EXERCISE LXXII. (Page 222.)

1. $x = 3$; $y = 6$.   2. $x = \frac{1}{2}$; $y = 1$.

3. $x = \frac{1}{2}$; $y = 1$.   4. $x = -2$; $y = \frac{2}{3}$.

5. $x = -\frac{2}{5}$; $y = \frac{1}{3}$.   6. $x = -\frac{1}{2}$; $y = 1$.

7. $x = y = a + b$.   8. $x = \dfrac{b(a^2 - bc)}{a - b}$; $y = \dfrac{c(a^2 - bc)}{a - c}$.

9. $x = -\frac{1}{78}$; $y = \frac{1}{132}$.   10. $x = y = \dfrac{a^2 + b^2}{ab}$.

EXERCISE LXXIII [a]. (Page 223.)

1. $x = y = z = 4$.   2. $x = 1\frac{5}{8}$; $y = 7\frac{1}{8}$; $z = -2\frac{3}{8}$.

3. $x = 14\frac{1}{2}$; $y = -1\frac{1}{2}$; $z = -3\frac{1}{2}$.   4. $x = 2$; $y = -3$; $z = -4$.

5. $x = 18$; $y = -122$; $z = -79$.   6. $x = 4$; $y = 3$; $z = 7$.

7. $x = 4$; $y = 5$; $z = 6$.   8. $x = -1$; $y = -2$; $z = 5$.

9. $x = 1$; $y = 2$; $z = 3$.   10. $z = -1\frac{31}{41}$.

11. $x = 3$; $y = 21$; $z = -3$. 12. $x = 3$.

13. $x = 6$; $y = 8$; $z = 10$.   14. $x = 6$; $y = -2$; $z = -3$.

15. $x = 3$; $y = 6$; $z = 8$.   16. $x = -5$; $y = 9$; $z = -8$.

17. $x = -4\frac{1}{9}$, $y = -4\frac{2}{3}$. $z = -3\frac{1}{108}$.

[b.]

1. $x = \frac{1}{2}$; $y = \frac{1}{6}$; $z = -\frac{1}{3}$. Divide through by xyz in each equation.

2. $x = 5$; $y = 1\frac{1}{4}$; $z = \frac{5}{13}$.

3. $x = 1\frac{7}{24}$; $y = -\frac{2}{3}$; $z = 2\frac{2}{3}$.

4. $x = \frac{1}{a}$; $y = \frac{1}{b}$; $z = \frac{1}{c}$.

5. $x = \dfrac{ab + ca - bc}{2a}$; y and z symmetrically.

6. $z = 3abc \div (c - a)(b + c)$, x and y symmetrically.

7. $x = a - b$; $y = b - c$; $z = c - a$.

8. $x = 2a \div (b + c - a)$; y and z symmetrically.

9. $x = 1 \div (a - b)(a - c)$; y and z symmetrically.

10. $z = \frac{580}{619}$.   11. $x = \frac{1}{2}$; $y = \frac{1}{3}$; $z = \frac{1}{4}$.

12. $x = a^2 - b^2$; $y = b^2 - c^2$; $z = c^2 - a^2$.

13. $z = \frac{1}{2}(a + b + c)(2a + b + c)$; x and y by symmetry.

14. $z = \dfrac{mbpnd + mapnf}{mbp + mna - npc}$; $x = \dfrac{pmn(acb + bdc)}{mapd + npfc - pmfb}$.

15. $(x + y + z)(a + b + c) = (a + b + c)^2$; $x + y + z = a + b + c$;

$$x = \frac{a^3 + b^3 + c^3 - (a + b)(b + c)(c + a) + 5abc}{a^2 + b^2 + c^2 - ab - bc - ca}.$$

EXERCISE LXXIV [*a*].   (Page 226.)

1. $x = 4$; $y = 9$; $z = 16$; $u = 25$.
2. $x = 3$; $y = 0$; $z = 5$; $u = 2$.
3. $x = 5$; $y = 3$; $z = 1$; $u = 4$.
4. $x = -5\frac{2}{11}$; $y = 16\frac{5}{11}$; $z = -5\frac{3}{11}$; $u = 11\frac{7}{11}$.
5. $x = 3$; $y = 2$; $z = -4$; $u = 5$.
6. $x = 0$; $y = -1$; $z = 2$; $v = -4$.
7. $x = -1$; $y = -2$; $z = -3$; $u = -4$; $t = -5$.
8. $x = 1$; $y = 2$; $z = 3$; $u = 4$.

[*b*.]

1. $x = a + b + c$; $y = a + b - c$; $z = a - b + c$; $t = b + c - a$.
2. $x = 6$; $y = -1$; $z = 3$; $u = 2$.
3. $x = \frac{1}{3}(a - 2b + c + d)$; y, z, and u by symmetry.
4. $x = \frac{1}{2}(a - b + c - d + e)$; y, z, u, and v by symmetry.
5. $x = \frac{1}{2}(2a - b - c - e + 2d)$; y, z, and u by symmetry.
6. $x = \frac{1}{2}(a + d)$; y, z, u, and v by symmetry.
7. $x = 30$; $y = 20$; $z = 42$; $u = 72$;   (y should by z in second equation.)
8. $x = a + b + c$; y, z, &c., by symmetry.
9. $x = \frac{1}{5}(a + b + c + d + e - 4f)$; y, z, &c., by symmetry.
10. Divide each side of every equation by xyz; $x = 1 \div b - c$; y and z by symmetry.

EXERCISE LXXV [*a*].   (Page 229.)

1. $x = 714285$; $y = 142857$.   2. $x = 40$; $y = 65$.
3. Willie 4;   Charlie 8.   4. $x = 1.234$; $y = 5.678$.
5. $x = 147$; $y = 63$.   6. 76.
7. 13 : 17.   8. 73.
9. 480 gallons;   400 gallons;   560 gallons.
10. \$10260;   \$7560.   11. $\frac{4}{6}$.
12. $10x + y = 6(x + y)$; $\therefore 4x = 5y$; $\therefore 10y + x = 9x = 5x + 5y$, etc.
13. 98 or 89.   14. A, 200 lbs.;   B, 250 lbs.;   C, 350 lbs.

15. 82 apples; gave away 2.   16. $5000; $3000; $4000.

17. 40; 88; 104.   18. 486.

19. $\dfrac{a(b-c)}{b-a}$; $\dfrac{b(c-a)}{b-a}$.   20. $x = \dfrac{2b}{1-a}$; $y = \dfrac{2b}{1+a}$.

21. $\frac{3}{4}$, $\frac{1}{4}$.

22. A, 105; B, $52\frac{1}{2}$; C, 210 minutes; A, B, and C in 30 minutes.

[b.]

1. First, 220 gallons; Second, 100 gallons.

2. 3674.   3. A, $40; B, $24; C, $16.

4. $d(m+n) \div 2mn$; $d(n-m) \div 2mn$.

5. $x+y : x-y : xy :: 5 : 1 : 18$; $x = 9$; $y = 6$.

6. $x = \dfrac{l(m-1)(q-mr) - (1-mn)(mp-lq)}{m^2 l(1-n) + mnl(1-m) - m(1-mn)}$.

7. 130.   8. 315 miles.   9. 9; $8\frac{2}{11}$ miles per hour.

10. $x = (p+1)n$; $y = (pq-1)n$; $z = (q+1)n$.

11. $x = 1\frac{5}{7}$; $y = 2\frac{2}{5}$; $z = -12$.

12. $\dfrac{c(a+b)}{2ab}$; $\dfrac{c(b-a)}{2ab}$.   13. $\dfrac{qrm}{qr-ps}$ hours.

14. A, $2.60; B, 1.26\frac{2}{3}$; C, 61$\frac{1}{3}$ cts.   15. *Ans.* $3\frac{1}{4}$ miles.

16. 3000 ft. from first station; x = distance from first station;

y = A's rate per second; z = B's rate;

then $\dfrac{4000}{y} - \dfrac{4000}{z} = 40$; $\dfrac{x}{y} - \dfrac{x}{z} = 30$;

substitute in this the value of $\dfrac{1}{y} - \dfrac{1}{z}$, viz. $\frac{1}{100}$, from first

equation and $x = 3000$.

17. Gold coin, $2; silver, $1.

18. $11\frac{1}{8}$ miles; 7 miles; and $5\frac{1}{8}$ miles.   19. $5200; $2480.

20. x = time for A, y for B, z for C;

then $\dfrac{1}{y} + \dfrac{1}{z} = \dfrac{m}{x}$, or $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = \dfrac{m}{x} + \dfrac{1}{x} = \dfrac{m+1}{x}$;

$\therefore m+1 = \dfrac{xy+yz+zx}{yz}$, similarly for $n+1$ and $p+1$.

21. x = rate of locomotive, y = rate of coach, z = distance;

then $6x + 8y = z = 5\frac{1}{2}(x + 1\frac{3}{4}) + 7\frac{1}{2}(y + 1\frac{3}{4})$

$= 7\frac{1}{12}(x - 1\frac{3}{4}) + 5\frac{1}{12}(y - 1\frac{3}{4})$; $x = 38\frac{1}{4}$, $y = 7$, $z = 287$.

After should be *before* in first line of equation.

22. Let $m,\ n$ be the required *dividends :*

then given fraction $= \dfrac{m}{3 + 4z} + \dfrac{n}{6 + 7z}.$

Multiply out and equate coefficients, and we get

$6m + 3n = 27$, $7m + 4n = 34$; $m = 2,\ n = 3$.

23. $\dfrac{bc - ad}{cf - de}$; $\dfrac{ab - be}{cf - de}$.   24. $\dfrac{9}{8x - 7}$; $\dfrac{6}{5x - 4}$; $\dfrac{3}{2x - 1}.$

25. 16, 20, 42.

EXERCISE LXXVI.   (Page 239.)

6. $-\frac{2}{3}$, -10; $\pm(a - b)$; $\pm(1 - a)$; $\pm a + \frac{3}{4}$.

7. $\pm(3a - 2b)$; $\dfrac{a + b}{a - b}$; 3, $\frac{5}{3}$.

8. $2b$, $-2a$; ± 123; ± 275.   10. ± 2; $\pm 2\frac{1}{2}$; $\pm 2\frac{1}{3}$.

12. ± 5; $\pm \frac{5}{3}a$; $\pm a$.   13. $\pm\sqrt{(mn)}$; $\frac{89}{63}$, $\frac{1}{7}$; ∞.

15. 12, 8; $\pm\sqrt{\left(\dfrac{a + c}{c - 1}\right)}.$

16. $\pm\sqrt{(ac)} \div \sqrt{(bd)}$; $\pm\sqrt{(ab)}$; ± 1.

17. ± 1; $2a$, $2b$.   18. $\pm\sqrt{(m^2 + n^2)}$; $\pm\sqrt{(a^2 + b^2)}$.

19. $\pm\sqrt{(a^2 + 2ab - 3b^2)}$; $\pm\sqrt{(a^2 + ab + b^2)}$.   20. ± 1.

EXERCISE LXXVII.   (Page 241.)

1. $11\frac{1}{2}$.   2. ± 385.   3. ± 13.

4. ± 77, ± 91.   5. $\pm\sqrt{10}$.   6. 78, 52, 39.

7. 481, 259.   8. $\pm\sqrt{3}$, 0.   9. 2, 4, 8.

10. $\pm\sqrt{6}$.   11. 15 or 21.   12. $a \pm \sqrt{(a^2 - b)}$.

13. 40 ft.   14. 8, 12, and -40, 60.   15. 12 ft.

16. 140, 120.   17. 20 per cent.   18. 960, 280.

19. 285, 152.   20. 4 hrs.

EXERCISE LXXIX. (Page 250.)

1. $-7, 1$; $6, 2$; $-7, -3$.
2. $-2, -1$; $-m \pm \sqrt{(m^2 + n)}$; $\frac{1}{2}\{m \pm \sqrt{(m^2 + 4n)}\}$.
3. $12, -2$; $-6, -20$; $30, -4$.
4. $23, -1$; $\frac{3}{2}, -\frac{1}{3}$; $\frac{1}{5}, -4$.   5. $2\frac{2}{5}, 1$; $-\frac{1}{4} \pm \frac{1}{4}\sqrt{-543}$.
6. $\frac{15}{13}, 9$; $\dfrac{51 \pm \sqrt{1721}}{40}$; $-5 \pm \sqrt{2}$.   7. $6\frac{1}{2}, -4\frac{67}{108}$; $3\frac{1}{4}, -2\frac{1}{4}$.
8. $4, \frac{4}{3}$; $2, \frac{32}{8}$; $28, -3$.   9. $\frac{32}{27}$; $-1\frac{1}{6}, 4$.
10. $-\frac{8}{15}, 2$; $-23, 1$.   11. $\frac{1}{2}(-5 \pm \sqrt{65})$; $-2a, a$.
12. $a(a - b), b(a + b)$; $-1, m \div (m - n)$.
13. $56\frac{7}{8}, 12\frac{3}{4}$; $(m + n)^2, -(m - n)^2$.   14. $a + b, a$; $a + b, 0$.
15. $a \pm b$; $-2a, 2b$; $3, a^2 \div (6 - a)$.

EXERCISE LXXX [a]. (Page 257.)

1. $4, 3$; $-\frac{2}{3}, 1$; $-\frac{7}{4}, 4$; $-2\frac{1}{2}, 2\frac{1}{3}$.
2. $-\frac{7}{4}, 4$; $-5, -\frac{8}{5}$; $3\frac{1}{2}, 5$.   3. $-\frac{3}{2}, 6$; $-\frac{5}{2}, 7$; $\frac{1}{3}, -2$.
4. $30, -4$; $6 \pm \sqrt{-64}$; $\frac{29}{6}, -\frac{5}{3}$.   5. a, b; b, a.
6. $c \div (a - b), d \div (a - b)$; $3, -5$.
7. $2, -3\frac{1}{2}$; $a + b, \dfrac{ab}{a + b}$.

[b.] (Page 258.)

Note.—*Imaginary roots generally omitted.*

1. $\pm \sqrt{2}$; $\pm 1, \pm 2$; $\pm 2, \pm 3$.
2. $\pm \sqrt{13}$; $\pm 2, \pm \sqrt{10}$; $-\frac{3}{7}, -\frac{6}{7}$.
3. $2, -\sqrt[3]{11}$; $2, 2\frac{1}{4}$.   4. $3, 2$; ± 11.   5. $-2, 4$; 2; 1.
6. $\pm \sqrt{\{+\frac{1}{2}(-3 \pm \sqrt{201})\}}$; $x^2 + 5 = 2$, or -1.
7. $(\pm \sqrt{2}) \div 4$; $-2a, a\sqrt[3]{2}$; $x^2 = \frac{1}{2}\{a^2 \pm \sqrt{(a^4 - 4b^4)}\}$.
8. $-1, 2$; $\pm \frac{3}{4}, \pm \frac{1}{4}\sqrt{11}$; $1, -2$.
9. $\frac{1}{2}\{-7 \pm \sqrt{41}\}, \frac{1}{2}\{-7 \pm \sqrt{21}\}$; $-4, 3, 3 \pm 21$.
10. $\frac{1}{2}\{1 \pm \sqrt{(4a - 3)}\}$; $1, 2, 4$.   11. -1; $-1, -2, 2\frac{1}{2}$; 1.
12. $x^3 - 8 + x^2 - 4 = 0, x = 2$; 3; $0, -1, -1$.
13. $1, \frac{1}{2}\{p - 1 \pm \sqrt{(p^2 - 2p - 3)}\}$; $1, \frac{1}{2}\{2p - 1 \pm \sqrt{(4p^2 - 4p - 3)}\}$; $1, x^2 + x + 1 = 0$.

14. $-1, \pm 2$; $a, -a, -b$; $0, \frac{1}{3}, -\frac{1}{2}$.

15. $1, 1, \frac{1}{2}(-3 \pm \sqrt{5})$; $2, \frac{1}{2}, -2 \pm \sqrt{3}$;

equation is $\left(x + \dfrac{1}{x}\right)^2 + 1\frac{1}{2}\left(x + \dfrac{1}{x}\right) - 10 = 0$, or

$2y^2 + 3y - 20 = 0$ $\left(\text{if } y = x + \dfrac{1}{x}\right)$, $(2y - 5)(y + 4) = 0$, etc.

[c.]

1. $7, -2$. 2. $\frac{3}{4}, 3$. 3. $2, -22$.

4. $a + 2c, -\dfrac{a(a+b)}{a+2c}$. 5. $\pm \sqrt{6}, \pm \sqrt{11}$. 6. $0, -\frac{23}{13}$.

7. $3, -1$. 8. $5, -4\frac{2}{7}$. 9. $0, \pm 6$. 10. $-2\frac{1}{2}$.

11. $\dfrac{\sqrt{\{(a-b)(c+d)\}} \pm \sqrt{\{(a+b)(c-d)\}}}{\sqrt{\{(a-b)(c+d)\}} \mp \sqrt{\{(a+b)(c-d)\}}}$. Use Art. 133.

12. $-\frac{3}{4}$. 13. $\pm 8, \pm 1$ (put y for $x^2 - 8$ in the denominators).

14. Complete the divisions, transpose and divide by 3, and

$\dfrac{x^2 - \frac{1}{2} - 9x - 3}{x + 3} = x - 9\frac{1}{2}$, $2\frac{1}{2}x = 25$, $x = 10$. 15. $4, 0$.

16. $(4a - 5b) \div 6ab$, $(a - 2b) \div 3ab$.

17. $\pm \frac{33}{65}\sqrt{-1}$, $\pm \frac{56}{65}\sqrt{-1}$.

18. Use first quantity for the *unknown*; $7, 1, 4, 4$. 19. $1, 3$.

20. $\pm 3\sqrt{\pm 1}$, $\pm 2\sqrt{\pm 1}$. 21. 2. 22. $c, c - \frac{1}{2}(a+b)$.

23. Complete the divisions, right member cancels, and

$\dfrac{1}{x-9} + \dfrac{1}{x-25} - \dfrac{2}{x-49} = 0$, or

$\dfrac{1}{x-9} - \dfrac{1}{x-49} = \dfrac{1}{x-49} - \dfrac{1}{x-25}$; $x = 19$.

24. Separate the factor $\dfrac{1}{x+a+b}$; $x = \dfrac{a^2+b^2}{a+b}$.

25. -1, and $n(x^2 - x + 1) + 1 = 0$.

26. $\frac{1}{2}\{-1 \pm \sqrt{45}\}$ ($+x$ should be $-x$ in given equation);

put equation in the form $(x^2 + x)^2 - (x^2 + x) = 132$, etc.

27. $\pm 2a, \pm 2a\sqrt{-1}$. 28. $2, 2\frac{1}{2}$. 29. $4, -\frac{5}{3}$. 30. $8, -\frac{8}{7}$.

[d.] (Page 260.)

1. $3, \frac{1}{3}, \pm \sqrt{-1}$ (see second equation in [b] 15).

2. Equation is $(x + 1)(x^2 - x + 1 \pm ax)$;

$x = -1, \frac{1}{2}\{1 - a \pm \sqrt{(a^2 - 2a - 3)}\}$.

3. $-1, -\frac{1}{2}, -2.$ 4. $2, -1, -4.$ 5. $-6, 4.$

6. $-2\frac{1}{2}, \infty.$ 7. $3, \infty.$ 8. $3, \frac{1}{3}, \frac{1}{2}(1 \pm \sqrt{-3}).$

9. Take together pairs of like numerators and equation reduces
to $x^4 - 5x^2 + 6 = 0 = (x^2 - 3)(x^2 - 2).$

10. $-4, -2, 3 \pm \sqrt{7}$; observe that one fraction is reciprocal of
the other; put it equal y, then $y + \dfrac{1}{y} = 2\frac{1}{2}.$ etc.

11. Use formula (4), p. 181, and factor by Ex. 7, p. 105;
$x = -2 \pm \sqrt{3}.$

12. $-\frac{1}{2} \pm \sqrt{\frac{69}{20}}.$ 13. $2 \pm \sqrt{2}$, and $x^2 - 6x + 6 = 0.$

14. $1 - p$, equation reduces to $(x + p - 1)\left(x^2 + x + \dfrac{1}{p-1}\right) = 0.$

15. $7, 2$; if $x^2 - 9x + 18 = y$, equation is $y^2 + 2y = 24$, etc.

16. Divide by left member and clear, $\therefore 63(1+x) = 62(1-x).$ etc.

17. $a\left(1 \pm 2\sqrt{\dfrac{b}{ac}}\right)$, or 0; complete the division in left member,
square, and 1 cancels.

18. $\frac{11}{12}$.   19. Sign before last fraction in left member should be $-$;
equation reduces to $7x^2 - 34x + 9 = 0.$

20. Write $-$ for $+$ before 50 in denominator; combine first pair
fractions and second pair; factor denominators and cancel;
then taking $x^2 - 5x = y$, $y^2 + 18y - 24 = 0$, etc.

21. $\frac{119}{110}$.   22. Combine first and last fractions, second and last
but one, etc., and $2x + 7$ is found to be a factor; then put
$x^2 + 7x = y$, and resulting equation is $y^2 + 18y + 90 = 0$;
$x = -3\frac{1}{2}$, etc.

23. Combine in pairs like numerators, then as in last example; $2\frac{1}{2}$.
$2\frac{1}{2} \pm \frac{1}{2}\sqrt{[5a + 13b + 17c \pm \sqrt{\{(a - 3b - 2c)^2 + 12ab\}}}$
$\div (a + b + c)].$

24. Combine first and second, etc.; $-\frac{1}{2}(a + b), \infty,$
$-\frac{1}{2}(a + b) \pm \frac{1}{2}\sqrt{\{\frac{1}{2}(a + b - 2c)^2 + \frac{1}{2}(a - b)^2\}}.$

25. See Ex. 8; $n, \dfrac{1}{n}. -n \pm \sqrt{(n^2 - 1)}.$

EXERCISE LXXXI [a].   (Page 264.)

1. 11 or -24.   2. 26 and 19.   3. $\frac{2}{2.5}$ or $-6\frac{1}{4}$.

4. $\frac{1}{2}(\sqrt{5}-1)a$, $\frac{1}{2}(3-\sqrt{5})a$.   5. $16\frac{1}{2}$.

6. 50 coffee, 60 raisins.   7. $240.

8. 100.   9. 12.   10. 11 vases.

11. A, 11 miles; B, 10.   12. 3, 4, and 5.

13. 25 cts.   14. $4d$. a dozen.   15. 3 and 18.

16. 7.   17. A, 72 miles; B, 54 miles.

18. $90 or $10.   19. $\frac{2\frac{1}{2}}{5}$.   20. A, $1800; B, $1600.

21. $3.   22. $\frac{1}{2}\{\sqrt{(2h^2-d^2)}+d\}$, $\frac{1}{2}\{\sqrt{(2h^2-d^2)}-d\}$.

23. 2.414 inches.

[b.]   (Page 266.)

1. 3 hours and 5 hours.   2. 36 and 30.   3. 63.

4. $-\frac{1}{4}(a+b)\pm\sqrt{\left(\dfrac{ab}{4m}+\dfrac{1}{16}(a+b)^2\right)}$.   5. 961.

6. 4200; read 780 in question.   7. 14 acres at $75.

8. 10 seconds.   9. $5\frac{5}{8}$ miles.   10. 5 miles an hour.

11. 15 miles.

12. If x be cost and s selling price, then $x=s+\dfrac{x^2}{100}$; on solving
    it is seen that $4s$ cannot be greater than 100;
    see Art. 175 (i).

13. $333\frac{1}{3}$, $666\frac{2}{3}$.

14. 72, 12, 8; Let $x^2=$ number remaining in smaller bag after
    handful is taken; then x^6 is left in *larger* bag, and $x^3=$
    number in handful, and x^4 is number in larger after
    second lot is taken out; then $x^4+x^2=\frac{5}{3}(x^2+x^3)$, and
    $x=2$, etc.

15. If x represents per cent, then $620=82x+(3790+82x)\dfrac{x}{100}$.
    $x=5$.

16. Let $2x=$ distance, then $\dfrac{x}{x-6}+\dfrac{x}{x-4}=\dfrac{6}{7}\cdot\dfrac{2x}{x-6}$; $x=22$.

17. Let x = rate backwards, $4x$ = rate forwards, then
$$\frac{\frac{3}{4}}{4x} + \frac{\frac{1}{4}}{x}, \; i.\,e.\; \tfrac{7}{16}x = \frac{\frac{3}{4}}{4x+2} + \frac{\frac{1}{4}}{x-\frac{1}{5}}; \; 1 \text{ mile an hour.}$$

18. 90.   19. 4900; x^2 being number of lines, equation is
$$\{\tfrac{49}{30}(x-10)\}^2 = 2\{x^2 - \tfrac{49}{30}(x-10)\}, \text{ or } 601x^2 - 20x\cdot2254$$
$$= -100\cdot49\cdot43; \; x=70.$$

20. A, half-past 4 o'clock; B, 5 o'clock.

EXERCISE LXXXII [a]. (Page 272.)

1. $x = \pm b,\; y = a \pm b.$ 2. $x = \pm b,\; y = a \mp b.$
3. $x = 3,\; -\frac{2}{5},\; y = 1,\; -1\frac{4}{5}.$ 4. $x = 3,\; -2\frac{1}{2},\; y = 3,\; -8.$
5. $x = 4,\; -10\frac{1}{4},\; y = 5,\; -12\frac{13}{16}.$ 6. $x = \pm 7,\; y = \pm 2.$
7. $x = \pm 20,\; y = \pm 16.$ 8. $x = \pm 15,\; y = \pm 3.$
9. $x = 30,\; 10,\; y = 10,\; 30.$ 10. $x = 1,\; -5,\; y = -1,\; -7.$
11. $x = 3,\; -3\frac{1}{4},\; y = 4,\; -3\frac{19}{28}.$ 12. $x = 2,\; -\frac{1}{3},\; y = 3,\; \frac{2}{5}.$
13. $x = 1,\; \frac{1}{7},\; y = 2,\; 2\frac{4}{7}.$ 14. $x = 10,\; 115,\; y = 6,\; -69.$
15. $x = 7,\; -4\frac{3}{4},\; y = 3,\; -2\frac{7}{8}.$ 16. $x = 1,\; -\frac{53}{81},\; y = -4,\; 8\frac{11}{27}.$
17. $x = \pm 7,\; \pm 5,\; y = \pm 11,\; \pm 9.$ 18. $x = 2,\; 3,\; y = 5,\; 4.$
19. $x = 2,\; 5,\; y = 6,\; 3.$ 20. $x = 5,\; \frac{3}{4},\; y = 3,\; -1\frac{1}{4}.$
21. $x = 7,\; 1,\; y = 3,\; 9.$ 22. $x = \pm\sqrt{13},\; y = \pm\sqrt{13}.$
23. $x = 3,\; y = 1.$ 24. $x = 2,\; \frac{5}{8},\; y = -7,\; -\frac{1}{8}.$

[b.]

1. $x = \pm 5,\; y = \pm 1.$ 2. $x = \pm 11,\; y = \pm 2.$
3. $x = 0,\; \pm 2,\; y = \pm\sqrt{\tfrac{1}{3}},\; \pm 1.$ 4. $x = 0,\; \pm 3,\; y = \pm 3,\; \pm 9.$
5. $x = \pm 3,\; \pm\frac{5}{3},\; y = \pm 5,\; \pm\frac{13}{3}.$ 6. $x = 0,\; \pm 1,\; y = \sqrt{\tfrac{7}{2}},\; \pm 1.$
7. $x = \pm 2\frac{1}{6},\; y = \pm\frac{1}{3}.$ 8. $x = \pm 3\sqrt{\tfrac{3}{7}},\; y = \pm\tfrac{1}{3}\sqrt{\tfrac{3}{7}}.$
9. $x = \pm 2\frac{1}{3},\; \pm 1,\; y = \pm 1,\; \mp 3.$ 10. $x = \pm 7,\; \pm 3,\; y = \pm 2,\; \pm 6.$
11. $x = \pm 1,\; \pm 1\frac{1}{2},\; y = \pm 5,\; \mp 1\frac{2}{3}.$
12. $x = \pm a \div \sqrt{(a+b)},\; y = \pm b \div \sqrt{(a+b)}.$
13. $x = \pm 6,\; y = \pm 2.$ 14. $x = \pm 3,\; \pm 8,\; y = \pm 5.$
15. $x = \pm 2,\; \pm\sqrt{\tfrac{2}{5}},\; y = \pm\frac{1}{2},\; \mp 2\sqrt{\tfrac{2}{5}}.$
16. $x = \pm 1,\; -3\sqrt{3},\; y = \pm 5,\; \pm\sqrt{3}.$
17. $x = 7,\; 1,\; y = 4,\; 7.$ 18. $x = 7,\; -5,\; y = 5,\; -7.$

19. $x = \pm 5, \pm 3, y = \pm 2, 7$.   20. $x = \pm 5, \pm 4, y = \pm 3$.

21. $x = 4, y = 3$.   22. $x = 14, 19, y = 19, 14$.

23. $x = 5, 4, y = 4, 5$.   24. $x = 4, 2, y = 2, 4$.

25. $x = 4, y = 3$.   26. $x = 3, 2, y = 2, 3$.

27. $x = 13, 9, y = 9, 13$.   28. $x = 7, 4, y = 4, 7$.

29. $x = 3, 1, y = 1, 3$.   30. $x = 3, 2, y = 2, 3$.

EXERCISE LXXXIII [a].   (PAGE 277.)

1. $x = 2, 1, y = -1, -2$.   2. $x = 4, -3, y = -3, -10$.

3. $x = \pm \frac{2}{3}, \mp \frac{1}{3}, y = \frac{4}{3}, 0$.   4. $x = \pm 1, y = \pm 7$.

5. $x = \pm 5, y = \pm 2$.   6. $x = \pm 1, y = \pm \frac{1}{2}$.

7. $x = \frac{1}{2}(a \pm 2b), \ y = \frac{1}{2}(a \mp 2b)$.

8. $x = \pm (a + b), \ y = \pm (a - b)$.

9. $x = \pm \dfrac{a^2 + b^2}{a - b}, \ y = \pm \dfrac{2ab}{a - b}$.

10. $x = \frac{1}{2}, y = \frac{1}{3}$.   11. $x = 6\frac{1}{2}, y = 1\frac{1}{2}$.

12. $x = 0, a, y = a, 0$.   13. $x = \pm 9, \pm 5, y = \pm 5, \pm 9$.

14. $x = \pm 7, \pm 3, y = \pm 3, \pm 7$.

15. $x = \pm 4, \pm 3, y = \pm 3, \pm 4$.

16. $x = 2, -1, y = 1, -2$.   17. $x = 11, y = 9$.

18. $x = -2, -3, 3 \pm \frac{1}{2}\sqrt{56}, \ y = -3, -2, 3 \mp \frac{1}{2}\sqrt{56}$.

19. $x = 5, -1, \frac{1}{2}(\pm\sqrt{41} + 5), \ y = 1, -5, \frac{1}{2}(\pm\sqrt{41} - 5)$.

20. Treat $x + y$ as the unknown; $x = \frac{1}{2}\{a \pm \sqrt{(a^2 - 48)}\}$,
    $y = \frac{1}{2}\{a \mp \sqrt{(a^2 - 48)}\}$, where $a = \frac{1}{2}(-3 \pm \sqrt{853})$.

21. $x = \frac{1}{2}(9 \pm \sqrt{-47}, \ y = \frac{1}{2}(9 \mp \sqrt{-47})$.

22. $x = 5, -2, -\frac{1}{2}(1 \pm \sqrt{41}), \ y = 2, -5$.

23. $x = 17, -6, \pm\sqrt{(118)} - 4, \ y = 3, -8\frac{1}{2}, 2 \pm \frac{1}{2}\sqrt{118}$.

24. $x = 5, 6, 7, 8, \ y = 8, 7, 6, 5$.

25. $x = \pm 13\sqrt{\frac{45}{218}}, y = \pm 7\sqrt{\frac{45}{218}}$.   26. $x = 4, 2, y = 2, 4$.

27. $x = 1, 10, y = 10, 1$.   28. $x = 3, 2, y = 2, 3$.

29. $x = 8, 4, y = 4, 8$.   30. $x = 1, 1\frac{7}{17}, y = 2, -\frac{1}{17}$.

31. $(a'c - ac')^2 + (ab' - a'b)(b'c - bc') = 0$.

EXERCISE LXXXIV. (Page 281.)

1. $x = 8, 2, \quad y = 4, \quad z = 2, 8.$ 2. $x = \pm 4, \pm 9, \quad y = \pm 6$, etc.

3. $x = 7, \quad y = 6, \quad z = 5.$ 4. $x = 1, \quad y = 2, \quad z = 3.$

5. $x = \frac{4}{5}, \quad y = \frac{4}{3}, \quad z = 4.$

6. $x = \pm \frac{4}{3} \sqrt{3}, \quad y = \pm \sqrt{3}, \quad z = \pm 2 \sqrt{3}.$

7. $x = 4, -7, \quad y = 3, -8, \quad z = 6, 28, -2z$ in text.

8. $x = 1, 9, \quad y = \pm 4, \quad z = 2, -6.$

9. $x = 2, -14, \quad y = 3, -15, \quad z = 4, -16.$

10. $x = 1, \quad y = -2, \quad z = 4.$ 11. $x = 4, \quad y = -5, \quad z = 7.$

12. $x = 2, 7, \quad y = 3, \quad z = 7, 2.$ 13. $x = y = z = \pm 1 \div \sqrt{2}.$

14. $x = 2abc \div (ac + bc - ab), \quad y, z$ symmetrical.

15. $x = \pm a^2 \div \sqrt{(a^2 + b^2 + c^2)}, \quad y, z$ by symmetry.

16. $x = \pm \sqrt{\{(a + b - c)(a + c - b) \div 2(b + c - a)\}},$
    y, z by symmetry.

17. $x = abc \div (ab + ac - bc), \quad y, z$ by symmetry.

18. $x = \sqrt{\{(c + a - b)(a + b - c) \div (b + c - a)\}}.$

19. Add n^2 to each equation and factor :
    $x = -n \pm (a + n)(c + n) \div (b + n), \quad y, z$ by symmetry.

20. $x = \pm a \sqrt{\{(b + c - a) \div [(a + b - c)(a - b + c)]\}},$
    y, z by symmetry.

21. $x = (bc + ca + ab) \div a, 0 ; \quad y, z$ by symmetry.

22. $x = 1, 2, 4. \quad y = 2, 4, 1, \quad z = 4, 1, 2.$

23. $x = \pm (-m + n + p) \div \sqrt{2(m + n + p)} ; \quad y, z$ by symmetry.

24. $x = \pm (-bc + ca + ab) \div \sqrt{(2abc)} ; \quad y, z$ by symmetry.

25. $x = 0,$ or $\pm 1 \div (c - a) ; \quad y, z$ by symmetry.

26. $x = \sqrt[3]{(b^2c^2 \div a)}; \quad y, z$ by symmetry.

EXERCISE LXXXV [a]. (Page 285.)

1. $\frac{1}{2}(1 + \sqrt{5}), \frac{1}{2}(3 + \sqrt{5}).$ 2. $\frac{1}{2} \pm \frac{1}{6}\sqrt{2193}, -\frac{1}{2} \pm \frac{1}{6}\sqrt{2193}.$

3. $36, 16,$ or $-36, -16.$ 4. $\pm \sqrt{(pq)}, \pm \sqrt{(p \div q)}.$

5. $20.$ 6. $\pm \frac{1}{2}(p + q)\sqrt{(a \div pq)}, \pm \frac{1}{2}(p - q)\sqrt{(a \div pq)}.$

7. $7, 21, 35.$ 8. $343, 64.$

9. $\frac{5}{7}, \dfrac{-1\frac{1}{2}}{\frac{1}{2}}.$ 10. $36.$

11. $34, 17, 51,$ or $-204, 612, -306.$ 12. $36.$

13. $\sqrt[3]{119} \pm \frac{1}{2}\sqrt{6} \div \sqrt{119}$.   Add and subtract the equations.

14. 8 ft., 10 ft.   15. 88 yds., 55 yds.

16. 63 ft., 45 ft.   17. 20 m., 30 m.

18. 6%, 7%.   19. 102 from length, 114 to width.

20. 18, 9·6.   21. 100 at $75 each.

22. 13, 10.   23. A 40 at $1.20, B 30 at $1.60.

24. 3, 5, 10.   25. 3, 4, 5.

[b.]

1. Edges (x, y, z) are 1, 2, 4 :
   $x + y + z = 7$, $x^2 + y^2 + z^2 = 21$, $x^3 + y^3 + z^3 = 73$.
   Cube first equation by formula H (3). p. 85, and substitute
   from third, second, and square of first.

2. 864.   3. 2, 5, 8. See Ex. LXXXV, 16.   Add first two
   equations and subtract third, then symmetry.

4. 76 ; the *one digit* remainder is of course 9.

5. $2145. $2\frac{1}{2}$ years.   6. 4, 7, 10.

7. 290 yds.   8. 48, 10.

9. 8, 9, 10; see Ex. 4, p. 284.

10. 342 ; in last line of problem read 29.

11. 12, 4. 3.   12. 3, 6, 9. See LXXXIV, 19.

13. $\frac{1}{2}\{b \pm \sqrt{(c - a^2)}\} : \frac{1}{2}\{a \pm \sqrt{(c - b^2)}\} :: \frac{1}{2}\{a \mp \sqrt{(c - b^2)}\}$
    $: \frac{1}{2}\{b \mp \sqrt{(c - a^2)}\}$.

14. $x = \sqrt{\{(1 + a)(1 + b) \div (1 + c)\}}$, y and z by symmetry.
    See 4, p. 284.

15. 28 workmen, each 45 lbs., or 36 workmen and each 77 lbs.;
    x = number of workmen, y lbs. carried each load, z number
    loads in one hour ; then $8xyz$ is whole weight moved, and
    $7(x + 8)(y - 5)z = 8xyz = 9(x - 8)(y + 11)z$.

16. $\dfrac{b - \sqrt{(b^2 - 4p)}}{a - \sqrt{(a^2 - 4p)}} = \dfrac{a + \sqrt{(a^2 - 4p)}}{b + \sqrt{(b^2 - 4p)}}$, where p = product of ex-
    tremes (or means), and $= (a^3 + b^3 - c) \div 3(a + b)$.

17. Of the first, 135, 62; of the second, 182, 57, (yds.).   18. 126.

EXERCISE LXXXVI [a]. (PAGE 296.)

1. $x^2 - 2x - 2 = 0$.   2. $\frac{2\,5}{2\,4}$.

3. $32x^2 - 1412x - 23205 = 0$.   4. $x^2 - a^3b = 0$.

5. $p^2 - q$; $p(p^2 - 3q)$; $(p^2 - 2q) \div q^2$; see Art. 178.

6. $x^2 - (4a - 6b)x + 9a^2 - 10ab + 8b^2 = 0$.   7. $3 \cdot 14159$.

8. Positive for *all* values of x, expression $= (x - 2\frac{1}{2})^2$.   9. $789\frac{10}{11}$.

10. $p - 2q + 3r$.   11. See Ex. 1, p. 294.

16. Assume $x^n = Ax + Bq$, then, since α, β are values of x,
     $\alpha^n = Ax + Bq$, and $\beta^n = A\beta + Bq$, whence A and B.

[b.]

2. $b^2 - ac = 0$.

4.   1. $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$;

     2. $q^2x^2 - (p^2 - 2q)x + 1 = 0$;

     3. $x^2 - (p^2 - q)x + q(p^2 - 2q) = 0$;

     4. $x^2 + \{p - \sqrt{(p^2 - 4q)}\}x - p\sqrt{(p^2 - 4q)} = 0$.

8. $cc' - aa' = 0$, $b'c + a'b = 0$.

[c.]

4. 579 and 135 are the roots of the first equation, 579 and -135 those of the second.

12. $4ab^2 + a'c - aa'c' = 0$; let roots of first equation be α, β, of second $\alpha + m$, $\beta + m$; form equations from relations of roots and coefficients and eliminate m.

13. (Right side of first equation should be 1.) Substitute for y in second equation, and apply condition of *equal roots* to resulting equation in x.

EXERCISE LXXXVII [a]. (PAGE 300.)

1. Min. 4.   2. Min. $-\frac{17}{4}$.   3. Max. $\frac{5}{4}$.

4. Min. $\frac{1}{18}$.   5. Min. $\frac{1}{56}$.   6. Min. $-\frac{1697}{2844}$.

7. Min. $\frac{7}{4}$.   8. Min. 2.   9. Min. $-\frac{1}{2}$, Max. $\frac{1}{2}$.

10. Max. 36 area, *i. e.* line is *bisected*.

[b.]

2. 81.   3. Min. $\frac{1}{2}a^2$, $i.\,e.$ line is bisected.

4. $\frac{1}{2}a\sqrt{2}$, the sides are equal.   6. $(a+b)^2 \div 4ab$.

7. *All* numbers between $\frac{1}{3}$ and 3.   8. $\frac{1}{2}\sqrt{1} + \frac{1}{2}$.

11. $(b^2 - 4ac) \div a^2 = (n^2 - 4mr) \div m^2$.   12. $p = 6$, or $\frac{2}{3}$.

EXERCISE LXXXVIII [a]. (PAGE 305.)

1. 1.   2. $24b^9$.   3. 5.   4. 1.

5. $\sqrt[3]{a^2}$; $2\sqrt[4]{a^3}$; $5a^3\sqrt{b}$; $7\sqrt{ab^3}$;

$6\sqrt{a}\sqrt[4]{b}$; $\sqrt{a}\sqrt[3]{b}\sqrt[4]{c}$; $\sqrt[4]{a^3}\sqrt[7]{b^6}\sqrt[3]{c^{10}}$.

6. $a^{\frac{1}{2}}$; $a^{\frac{2}{3}}b^{\frac{4}{3}}$; $a^{\frac{1}{2}}b^{\frac{3}{4}}c^{\frac{1}{5}}$; $a^{\frac{2}{7}}b^{-\frac{3}{7}}c^{\frac{1}{7}}$; $a^{\frac{6}{5}}b^{\frac{2}{5}}c^{-\frac{1}{5}}d^{\frac{1}{20}}$.

7. *1.* a^2b^{-3}; $a^{-1}b^{-1}c$; $a^{\frac{1}{2}}b^{-2}$; $7a^{\frac{5}{2}}b^{\frac{3}{4}}$; $a^{-2}b^{-2}$; a^2b^2;

$5a^5b^7c^{-5}$; $6a^{\frac{5}{4}}b^{\frac{1}{6}}c^{-\frac{7}{8}}$.

$\quad$ *2.* $\dfrac{1}{a^{-2}b^3}$; $\dfrac{1}{abc^{-1}}$; $\dfrac{1}{a^{-\frac{1}{2}}b^2}$; $\dfrac{7}{a^{-\frac{5}{2}}b^{-\frac{3}{4}}}$; $\dfrac{1}{a^2b^2}$; $\dfrac{1}{a^{-2}b^{-2}}$;

$\dfrac{5}{a^{-5}b^{-7}c^5}$; $\dfrac{6}{a^{-\frac{5}{4}}b^{-\frac{1}{6}}c^{\frac{7}{8}}}$.

[b.]

1. $(a^2 - b^2)^n$; $(x+y)^{p-q}$; $(x-y)^n$.   2. $\frac{3}{2}a^{\frac{45}{4}}$; 1.

3. $x^{\frac{4}{3}} - 4x^{\frac{1}{2}}$; $a^{\frac{2}{3}}b - a^{\frac{5}{4}}b^2 + 3a^2b^{\frac{4}{5}}$;

$ab^{\frac{8}{3}} + a^{\frac{3}{2}} - a^2b^{\frac{1}{3}} - a^{\frac{6}{5}}b^{\frac{4}{5}}$.

4. $a^{\frac{1}{3}}b^{\frac{4}{3}} - 5ab^{\frac{6}{5}} + a^{\frac{4}{3}}b^{\frac{4}{3}} - a^{\frac{14}{5}}b^{\frac{16}{5}}$; $a^{-\frac{11}{5}}$; $3y^{-\frac{2}{3}}$; $x^{\frac{43}{20}}y^{\frac{5}{2}}$.

5. $\dfrac{c}{a^2b^3} - \dfrac{a}{b^3} + \dfrac{b}{a^4c^3} - \dfrac{1}{ab^2c^3}$; $\dfrac{c}{a^{\frac{2}{3}}b^{\frac{1}{3}}} + \dfrac{b^{\frac{2}{3}}c^{\frac{1}{3}}}{a^{\frac{2}{3}}} + \dfrac{1}{b^{\frac{3}{4}}}$.

6. $\dfrac{c^3}{a^2b^3} + 2abc - \dfrac{3a^3}{b^2c^3} + a^2b^2c^2$; $\dfrac{x^2c^4}{a^3y^2} - \dfrac{x^{a+b}}{y^{a+b}} + \dfrac{y^4}{5x}$.

7. $\sqrt[5]{x^3} + \sqrt[3]{y^2} - \sqrt[5]{z^3}$; $\sqrt[3]{x^{-2}} \cdot \sqrt[4]{y^3}$; $21\sqrt{a^{-6}}$;

$\dfrac{3\sqrt[3]{b^2}}{4\sqrt[4]{a^5}}$.

8. $\dfrac{\sqrt[3]{c}}{\sqrt[3]{a^3b^2}} + \dfrac{\sqrt{a^3}}{\sqrt[3]{b^2}} - \dfrac{\sqrt[3]{a^2}}{\sqrt[4]{b^3}} + \dfrac{\sqrt{b}}{\sqrt{a}}$;

$\sqrt[2n]{x} \cdot \sqrt{a^3}$; $\sqrt[12]{a^x}$; $\sqrt[n]{a^{11}}$.

9. 8 ; $\frac{1}{32}$; $\frac{1}{25}$; $\frac{1}{16}$; $\frac{1}{11}$; $\frac{1}{9}$. 10. $1024a$; $a \div 32$.

11. $\dfrac{625a^5}{3840b^2}$; $a^{6x-8y} \cdot b^{10x-12y} \cdot c^{14x-16y}$. 12. $(7x - 6y)^{\frac{4}{3}}$; $(5a - 7b)^x$.

13. 244140625 ; 2. 14. 0.

[c.] (Page 306.)

1. $\dfrac{b^{\frac{2}{9}}}{a^{\frac{5}{9}}}$; $a^{\frac{3}{4}}b^{\frac{4}{3}}$; $\dfrac{81x^8}{16a^8}$; $x^{\frac{5}{12}}$. 2. x^{2n-1} ; $a^{\frac{5}{4}}b^{\frac{3}{8}}$; $\dfrac{1}{(x-y)^2}$.

3. $a^{14}b^{-23}$; $a^{\frac{1}{9}}b^{-\frac{4}{9}}$; $\left(\dfrac{x}{y}\right)^7$. 4. $a^2b^{\frac{m2+n2}{mn}}$; $b^{\frac{m2-n2}{mn}}$; a^{11mnp} ; b^{m3-mn}.

5. $a^{-\frac{1}{4}}$; $a^{12}b^6$; $x^{-4}y^4$; $a^{\frac{m2+n2}{mn}} \cdot b^{\frac{m+n}{n}}$. 6. $a^6b^4c^2$; a^2b^6c.

7. $\left(\dfrac{a^{\frac{5}{2}}b^2}{c^{\frac{3}{2}}}\right)$; $-\dfrac{x^6}{y^6}$; $a^{\frac{49}{36}}b^{\frac{37}{36}}$.

EXERCISE LXXXIX [a]. (Page 309.)

1. x^{a+b+c} ; x^{a2+a+1} ; x^{m3n2} ; $a^{\frac{m3}{m+1}}$.

2. 2^{n2} ; $\frac{1}{b}$; $(a^{\frac{1}{n}} \cdot b^{-\frac{1}{m}})^{pq}$.

3. $x^{\frac{9}{2}} + x^4 + x^{\frac{7}{2}} + 2x^3 + x^{\frac{5}{2}} + x^2 + x^{\frac{3}{2}} + x + 1$; $x + y$.

4. $x^{\frac{3}{2}} - y^{\frac{3}{2}}$; $x^2 + y^2$.

5. $x^4 + 2x^2y^2 + y^4 - xy$; $x^2 - 2xy + y^2$. 6. $4a^2 - b^2$.

7. $2x^{2a} - 42 - 9x^a + 6x^{-2a} + 11x^{-a}$; $4x^{\frac{2m}{n}} - 9y^{\frac{2p}{q}}$.

8. $a^2 + a^{-3} - 2 - a^{\frac{4}{3}} - a^{-\frac{2}{3}} + 2a^{\frac{1}{3}}$.

9. $x^3 + 4x^2y^{\frac{2}{3}} - 4x^2y - 16xy^{\frac{4}{3}} + 16xy^{\frac{5}{3}} - 64y^2$.

10. $3 + 2x^{-\frac{n}{2}} + 2x^{\frac{n}{2}} + x^{-n} + x^n$; $a^{\frac{4}{3}}x^{\frac{4}{3}} - b^4$.

11. $x^{\frac{1}{3}} - y^{\frac{1}{3}}$; $x + y + x^{\frac{1}{3}}y^{\frac{2}{3}} + x^{\frac{2}{3}}y^{\frac{1}{3}}$.

12. $x^{\frac{4}{5}} - x^{\frac{3}{5}}y^{\frac{1}{5}} + x^{\frac{2}{5}}y^{\frac{2}{5}} - x^{\frac{1}{5}}y^{\frac{3}{5}} + y^{\frac{4}{5}}$; $8a^{-2} + 7a^{-1} + 6$.

13. $5b^{\frac{1}{2}} + 4b^{\frac{1}{6}} + 3b^{-\frac{1}{6}} + 2b^{-\frac{1}{2}}$.

14. $x^{-\frac{5}{3}} + x^{-\frac{4}{3}}y^{-\frac{1}{3}} + x^{-1}y^{-\frac{2}{3}} + x^{-\frac{2}{3}}y^{-1} + x^{-\frac{1}{3}}y^{-\frac{4}{3}} + y^{\frac{5}{3}}$;

$\quad a^{-\frac{5}{2}} - 2a^{-2}b^{\frac{1}{3}} + 4a^{-\frac{3}{2}}b^{\frac{2}{3}} - 8a^{-1}b + 16a^{-\frac{1}{2}}b^{\frac{4}{3}} - 32b^{\frac{5}{3}}$.

15. $x^{\frac{4}{3}} - 4x + 10x^{\frac{2}{3}} - 16x^{\frac{1}{3}} + 19 - 16x^{-\frac{1}{3}} + 10x^{-\frac{2}{3}} - 4x^{-1} + x^{-\frac{4}{3}}$.

16. $(a^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})(x^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})$;

$\quad (x^{\frac{2}{3}} - 2x^{\frac{1}{3}}y^{-\frac{1}{3}} + 2y^{-\frac{2}{3}})(x^{\frac{2}{3}} + 2x^{\frac{1}{3}}y^{-\frac{1}{3}} + 2y^{-\frac{2}{3}})$.

17. $(x^{\frac{1}{2}} - 8)(x^{\frac{1}{2}} + 7)$; $(3x^{\frac{3}{4}} - y^{\frac{1}{2}})(3x^{\frac{3}{4}} + 2y^{\frac{1}{2}})$.

18. $(x - 1)(x - x^{\frac{1}{2}} + 1)$; $(3x^{\frac{1}{2}} - 2y^{\frac{1}{2}})(2x^{\frac{1}{2}} - 3y^{\frac{1}{2}})$.

19. $-a^{-1}(1 + b^{-1})$, b^{-1} in denominator; $\left(\dfrac{a}{b}\right)^{\frac{1}{2}(m+1)}$;

$$\frac{a^{\frac{2}{3}} + b^{\frac{2}{3}} + c^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} - b^{\frac{1}{3}}c^{\frac{1}{3}} - c^{\frac{1}{3}}a^{\frac{1}{3}}}{a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}}}.$$

20. $ab^{-1} + 1 + a^{-1}b$.   21. $a^{\frac{2}{5}} + 2a^{\frac{1}{5}}y^{\frac{1}{5}} + 3y^{\frac{2}{5}}$.

[b.] (Page 310.)

1. 0;   expression $= (ab - a^{-1}b^{-1})(ab + a^{-1}b^{-1}) - ab(ab^{-1} + a^{-1}b)$
$+ a^{-1}b^{-1}(a^{-1}b + ab^{-1}) = (ab - a^{-1}b^{-1})(ab + a^{-1}b^{-1}) - (ab^{-1} + a^{-1}b)$
$\times (ab - a^{-1}b^{-1}) = (ab - a^{-1}b^{-1})(ab + a^{-1}b^{-1} - ab^{-1} - a^{-1}b) = 0$.

2. See last question.

3. For last term read $a^{\frac{5}{6}}$.   *Ans.* $a^{\frac{1}{3}} - 2a^{\frac{1}{2}} + a^{\frac{5}{6}}$.

4. $(x^{47}y^{46})^{\frac{1}{6}}$.   5. $2x(3 + x^2) \div (1 + x)^3$.

6. $a^{\frac{4}{3}} - 2ab^{\frac{1}{2}} + 3a^{\frac{2}{3}}b - 2a^{\frac{1}{3}}b^{\frac{3}{2}} + b^2$;   in divisor read $2a^{\frac{1}{3}}b^{\frac{1}{2}}$.

7. $x^4 + 2x^3 - 8x^2 - 6x - 1$.

8. $ax^3 - a^{\frac{1}{3}}x = a^{\frac{1}{3}}x(a^{\frac{2}{3}}x^2 - 1)$, and second factor of this is con-
tained in the product of the other two quantities,
$\quad \therefore$ L. C. M. $= a^{\frac{1}{3}}x(a^2x^6 - 1)$.

9. $ab^{-1} + 1 - \frac{1}{2}a^{-1}b$;   in second term in text read b^2 for b^{-2}.

10. $\frac{7}{12}$;   in first term read $a^{\frac{3}{2}}$.

11. $\dfrac{(x - a^{\frac{1}{2}})(x + a)}{x - a}$;   in second term of numerator read a.

12. $\sqrt[12]{(x^5y^6)} \div \sqrt[5]{(a^2b^2)}$.

13. Expression $= a^{\frac{2}{3}} (a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{1}{2}} + b^{\frac{2}{3}} (a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{1}{2}} =$ etc.

14. Numerator $= x^2 \left(x^{\frac{m-n}{n}} - x^{\frac{n-m}{n}} \right)$,

    Denominator = same factor $\times \left(x^{\frac{m}{n}} + x^{\frac{n}{m}} \right)$.

15. $a^{6x} + a^{5x-y} + a^{4x-2y} + a^{3x-3y} + a^{2x-4y} + a^{x-5y} + a^{-6y}$.

16. $abcde$; divisor $= d^{19}$ (say) $= d^{17} \times d^2$, apply Law IV.

17. $x^{\frac{1}{2}} + (1 - m) a^{\frac{1}{2}} x^{\frac{1}{4}} + a$.

18. Expression $= x^2 - y^2 - (y^{-2} - x^{-2}) = xy (xy^{-1} - x^{-1}y)$
    $- x^{-1}y^{-1} (xy^{-1} - x^{-1}y)$, etc.

19. $a^{\frac{1}{2}}x + 2$; second remainder is $- 5 (ax^2 + 7a^{\frac{1}{2}}x + 10)$
    $= -5 (a^{\frac{1}{2}}x + 2)(a^{\frac{1}{2}}x + 5)$, and H. C. F. is $a^{\frac{1}{2}}x + 2$.

20. $(ax^2 - 1) (ax^2 + 1) (a^3x^6 - 1) = a^5x^{16} - a^3x^6 - a^2x^4 + 1$.

21. $1 - \frac{1}{64} a^{-3} x^6$.

22. $a^{\frac{1}{3}}x (a^{\frac{1}{3}}x - 1) \div (2a^{\frac{1}{3}}x - 1$; read x^3 in first term of numerator;
    denominator $= (3a^{\frac{1}{3}}x + 1) (2a^{\frac{1}{3}}x - 1)$;
    $(1 - a^{\frac{3}{2}}) \div \{ a (5a + a^{\frac{1}{2}}) \}$;
    denominator $= a^{\frac{3}{2}} (a + a^{\frac{1}{2}}) (5a + a^{\frac{1}{2}})$.

23. $x - y + z = - \left\{ \dfrac{1}{(b-c)(c-a)} + \dfrac{1}{(a-b)(b-c)} + \dfrac{1}{(a-b)(c-a)} \right\}$
    $= 0$;
    Expression $= \dfrac{-(a-b)(b-c)(c-a)}{(a-b)(b-c)(a-c)} = 1$.

24. $(a + b) \dfrac{m^2 + 3n^2 + m - n}{2mn}$.

EXERCISE XC. (Page 313.)

1. $\sqrt[3]{x^2}$; $7\sqrt{(xy^3)}$; $5\sqrt[4]{(x^3y^7)}$; $6\sqrt[6]{(x^9y^4)}$; $\sqrt{(a^3b^4)}$.

2. $27^{\frac{1}{4}}$; $512^{\frac{1}{5}}$; $9^{-\frac{1}{3}}$; $(\frac{3}{2})^{\frac{1}{4}}$; $8^{\frac{1}{5}}$.

3. 1. $-\frac{2}{3} (b^6)^{\frac{1}{3}}$; $\frac{3}{4} (a^6b^9)^{\frac{1}{3}}$; $(4^{-9})^{\frac{1}{3}}$; $\{(\frac{3}{10})^3\}^{\frac{1}{3}}$; $(a^3b^{-3}c^{-9})^{\frac{1}{3}}$.
   2. $-\frac{2}{3} (b^{-8})^{-\frac{1}{4}}$; $\frac{3}{4} (a^{-8}b^{-12})^{-\frac{1}{4}}$; $(4^{12})^{-\frac{1}{4}}$; $\{(\frac{10}{3})^4\}^{-\frac{1}{4}}$; $(a^{-4}b^4c^{12})^{-\frac{1}{4}}$.

4. $\sqrt{(36)}$; $\sqrt[3]{(250)}$; $\sqrt{\frac{1}{2}}$; $\sqrt{6}$; $\sqrt[3]{(90)}$; $\sqrt[3]{(16)}$; $\sqrt[4]{\frac{1}{3}}$;
    $\sqrt[3]{a^4}$; $\sqrt[4]{(a^2b^5)}$.

5. $\sqrt{(ab)}$; $\sqrt{\dfrac{a}{b}}$; $\sqrt[3]{\dfrac{a^2}{b^2}}$; $\sqrt{(6a^3x)}$; $\sqrt[3]{\left(\dfrac{16a^2}{81b^2}\right)}$; $\sqrt[3]{\left(\dfrac{16a}{3}\right)}$;

   $\sqrt{(a^2 - b^2)}$.

6. $\sqrt[n]{(a^m b)}$; $\sqrt[n]{(a^{n+1})}$; $\sqrt[n]{\{(a^2 - x^2)^n (a + x)\}}$;

   $\sqrt{\left(\dfrac{a + b}{c}\right)^3}$; $\sqrt[3]{\left(\dfrac{x - 3}{x + 4}\right)^2}$.

7. $3\sqrt{10}$; $5\sqrt{5}$; $3\sqrt[3]{5}$; $9\sqrt{6}$; $18\sqrt[3]{2}$; $\frac{3}{2}\sqrt[3]{12}$; $7\frac{1}{2}$.

8. $8\sqrt[3]{2}$; $6\sqrt[5]{48}$; $2\sqrt[4]{5}$; $2\sqrt[4]{3}$; $10\sqrt{3}$; 2; $2\sqrt[3]{18}$;

   12; $ab\sqrt[n]{b}$.

9. $\frac{2}{3}\sqrt{2}$; $\frac{2}{27}\sqrt{2}$; $\frac{2}{27}\sqrt[5]{16}$; $ay^{\frac{3}{4}}$; $a\sqrt[n]{a^m}$; $a^2x\sqrt[3]{(ax^2 - 1)}$.

10. $\frac{3}{2}\sqrt[3]{150}$; $\sqrt[4]{375}$; $a^{\frac{1}{2}}(x + 5)$; $(x + y)\sqrt[3]{(x - y)}$.

11. $(x - a)\sqrt[3]{\{(x + a)(x^2 - a^2)\}}$; $x^{\frac{1}{2}}(x + y)$; $2(a - b)\sqrt[3]{(ab)}$.

12. $10\sqrt{3}$, $\frac{7}{2}\sqrt{3}$, $\frac{9}{15}\sqrt{3}$, $\frac{1}{2}\sqrt{3}$, $\frac{1}{6}\sqrt{3}$.

13. $4^{\frac{1}{2}}$, $3^{\frac{1}{2}}$; $8^{\frac{1}{3}}$, $6^{\frac{1}{3}}$; $10{,}000^{\frac{1}{4}}$, $1000^{\frac{1}{4}}$; $33^{\frac{1}{5}}$, $32^{\frac{1}{5}}$; $80^{\frac{1}{3}}$, $50^{\frac{1}{3}}$;

   $a^{\frac{4}{5}}$, $a^{\frac{3}{5}}$; $a^{\frac{4}{24}}$, $a^{\frac{3}{24}}$; $a^{\frac{49}{35}}$, $a^{\frac{25}{35}}$.

14. $2\sqrt[3]{3} = 24^{\frac{2}{6}}$; $3\sqrt{2} = 18^{\frac{3}{6}}$; $24^{\frac{2}{6}} = 576^{\frac{1}{6}}$; $18^{\frac{3}{6}} = 5832^{\frac{1}{6}}$;

   and $\frac{5}{2}\sqrt[4]{1} = (244\frac{9}{64})^{\frac{1}{6}}$.

EXERCISE XCI. (Page 317.)

1. $2\sqrt{2}$; $8\sqrt[3]{5}$.   2. $-12\frac{1}{2}\sqrt{3}$; $11\frac{2}{3}\sqrt[3]{9}$.

3. $60\sqrt{3}$; $80\sqrt{3}$; 24.   4. $6 - 5\sqrt{6}$; $6\sqrt{3} + 3\sqrt{30}$.

5. -32.

6. $\frac{1}{3}\sqrt{2} + \frac{1}{3}\sqrt{3} + 2\sqrt{5}$; $\frac{1}{3}\sqrt{6} + \frac{1}{2}\sqrt[6]{32} + \frac{1}{6}\sqrt[4]{120}$.

7. $\frac{3}{4}(\sqrt{7} + \sqrt{3})$; $\frac{1}{2}(7 + 3\sqrt{5})$.

8. $\frac{1}{2}(17 - 3\sqrt{5}$; $\frac{1}{4}(16 - 13\sqrt{2})$;

   $\frac{1}{11}(7\sqrt{14} - 13)$; $2a^2 - 1 + 2a\sqrt{(a^2 - 1)}$.

9. $288\sqrt[12]{72}$; see 4, p. 316.

10. $x + y + z + 2\sqrt{(xy)} - 2\sqrt{(xz)} - 2\sqrt{(yz)}$;

   $13x^2 + 4 + 12x\sqrt{(x^2 + 1)}$.

11. $\sqrt{x} - \sqrt{a}$; $x^{\frac{2}{3}} - (xa)^{\frac{1}{3}} - a^{\frac{2}{3}}$; $a - \sqrt{(ab)} + b$;

   $\frac{1}{79}(25 - 6\sqrt{2})(3 - \sqrt[4]{2})$.

12. Square and transpose radicals, square again, then
$$(ax + by + cz)^2 = 4(abxy + bcyz + acxz)$$
$$+ 8\sqrt{(abcxyz)} \{\sqrt{(ax)} + \sqrt{(by)} + \sqrt{(bz)}\}, \text{ etc.}$$

13. Rationalize.   14. 3.1003.   15. 3.160.

16. $(\sqrt{5} + 1) \{4 - \sqrt{(10 + 2\sqrt{5})}\} \div 4$; $\{a + \sqrt{(a^2 - x^2)}\} \div x$.

17. $4x\sqrt{(x^2 - 1)}$; $1 \div (1 - x^2)$.   18. $2x^2 \div a^2$.

19. a; rationalize and substitute.

20. $\sqrt{(a - x)} \div (\sqrt{a} + \sqrt{x})$:   factor out $\sqrt{(a + x)}$ in denominator of first fraction and rationalize, resulting numerator cancels denominator of last fraction, etc.   20. $10\frac{2}{3}$.

EXERCISE XCII [a]. (Page 320.)

1. $\sqrt{3} + \sqrt{2}$.   2. $\sqrt{11} + \sqrt{2}$.   3. $\sqrt{10} - \sqrt{6}$.

4. $2 + \sqrt{2}$.   5. $\sqrt{11} + \sqrt{5}$.   6. $\sqrt{5} + \sqrt{2}$.

7. $\sqrt{6} + 1$.   8. $2 + \sqrt{5}$.   9. $2\sqrt{3} + 3\sqrt{5}$.

10. $\dfrac{\sqrt[4]{7}}{\sqrt{2}}(\sqrt{7} - \sqrt{3}$.   11. $\sqrt{7} - \sqrt{3}$.

12. $3 - \sqrt{3}$; change 13 to 11.

13. $2\sqrt{5} - 3$.   14. $3\sqrt{11} - \sqrt{41}$.   15. $\sqrt{7} - \sqrt{2}$.

[b.]

1. $\sqrt[4]{3}(1 + \sqrt{2})$.   2. $\sqrt[4]{5}(1 + \sqrt{2})$.   3. $\sqrt[4]{2}(\sqrt{3} - \sqrt{2})$.

4. $5 + \sqrt{6}$.   5. $\sqrt{51} - 7$.   6. $\sqrt{17} + \sqrt{19}$.

7. $\sqrt[4]{6}(1 + \sqrt{2})$.   8. $\sqrt[4]{2}(\sqrt{3} - \sqrt{2})$.   9. $3\sqrt{5} + 6\sqrt{2}$.

10. $\frac{1}{2}(\sqrt{3} - \sqrt{\frac{3}{2}})$.   11. $\sqrt{30} - \sqrt{\frac{1}{2}}$.   12. $\sqrt[4]{2} + \sqrt[4]{\frac{1}{2}}$.

13. $\sqrt[4]{3}(1 + \sqrt{2})$.   14. $\sqrt{(ab - ab^2)} - \sqrt{(ab^2)}$.

15. $\dfrac{\sqrt{1 + x} + \sqrt{1 - x}}{2}$.   16. $3\sqrt{m} - 5\sqrt{n}$.

17. $\sqrt{(m^2 - n^2)} + n$.   18. $\sqrt{(x + y)} + \sqrt{(x - y)}$.

19. $\sqrt{x + y} + \sqrt{z}$.   20. $1 - x + \sqrt{1 + 2x - x^2}$.

21. $1 + \sqrt{2}$.   22. $\sqrt[4]{2}\,(\sqrt{5} + \sqrt{3})$; in text 60 under root sign.

23. $\sqrt{5} - 1$.   24. $\sqrt{5} + \frac{1}{2}\sqrt{3}$; 48 for 49 in text.

25. $\sqrt{3} - \sqrt{2}$.   26. $\frac{1}{2}(\sqrt{10} + \sqrt{2})$.

EXERCISE XCIII [a]. (PAGE 323.)

1. $a^2 \div 2$.   2. 8.   3. $\sqrt{5}$.   4. 21.

5. 9.   6. 2.   7. a^m.   8. 16.

9. -1.   10. a.   11. 25, $\frac{1}{25}$, read $5\frac{1}{5}$ in text.

12. 3.   13. 9.   14. 4 or $-14\frac{2}{3}$.   15. $7\frac{1}{4}$ or 4.

16. 363.   17. 9.   18. 2.   19. 2.

20. 5.   21. 6.   22. $3\frac{1}{2}$ or 16.   23. -243 or 32.

24. $\dfrac{a + b}{a - b}$.   25. 9.   26. $-1\frac{5}{16}$.

[b.]

1. $\dfrac{1}{b - 2}$.   2. $\dfrac{(a - b)^2}{2a}$.   3. -1.   4. 16.

5. $-(a + b)$.   6. 46.   7. 6.   8. $\frac{1}{3}$.

9. $\frac{1}{4}$. In text remove parentheses from denominator.

10. 27.   11. $\dfrac{(a - b)^2}{2a - b}$.

12. $\frac{1}{51}$.   13. ± 3; use $\sqrt{(x^2 + 7)}$ as the unknown.

14. 1.   15. $-1, 2, 3, 6$.   16. $\sqrt[4]{\frac{1}{4}(4a^2b^2 - b^4)}$.

[c.] (PAGE 324.)

1. $-2, 4, 6, 12$.

2. $0, \pm\sqrt{3}$; simplifies to $x\{\sqrt{(2 + x)} + \sqrt{(2 - x)}\}$

$$= \sqrt{2}\{2 + \sqrt{(4 - x^2)}\} = \frac{\sqrt{2}}{2}\{4 + 2\sqrt{(4 - x^2)}\}$$

$$= \frac{\sqrt{2}}{2}\{\sqrt{(2 + x)} + \sqrt{(2 - x)}\}^2; \quad \therefore \ \sqrt{(2 + x)} + \sqrt{(2 - x)}$$

is a factor, etc.

3. $0, (b^2 - 4a^2) \div 4a$.

4. $a(b - c) \div 2\sqrt{(bc)}$; see Ex. 5, p. 322.

5. $\dfrac{1}{a}\left(b - \dfrac{nc}{n-1}\right)^2.$
       6. $0,\ \pm\sqrt{3}.$

7. $\frac{8}{3}.$
       8. $81 \div a.$

9. $2a^2 \div (1 + a^2).$
       10. $-(a^2 + b^2) \div (a + b).$

11. $(1 + 4b - 10b^2 + 4b^3 + b^4) \div (1 + b)^4.$

12. $\dfrac{1}{a}\sqrt{\left(\dfrac{2a}{b} - 1\right)}$; equation is $\sqrt{\left(\dfrac{1 + bx}{1 - bx}\right)} = \dfrac{1 + ax}{1 - ax}$;

     square and use formula (6), p. 181.

13. $30.$
     14. $\left\{\dfrac{a^2(c-d)^2 - b^2(c+d)^2}{2(c^2+d^2)}\right\}^{\frac{1}{2}}.$
     15. $\frac{7}{9}.$

16. $\left\{\dfrac{(c^3 - 2a)^3 + 27a^3c^3}{27c^3}\right\}^{\frac{1}{2}}$; cube by formula G (2), p. 85.

17. $\pm\frac{1}{2}\sqrt{\left\{1 - \left(\dfrac{c-2}{3c^{\frac{1}{3}}}\right)^3\right\}^{\frac{1}{2}}}.$

18. $-\frac{781}{1267},\ -\frac{1023}{1025}$; divide by right member, and

     $\left(\dfrac{1+x}{1-x}\right)^{\frac{1}{5}} + \dfrac{3}{16}\left(\dfrac{1-x}{1+x}\right)^{\frac{1}{5}} = 1,$ or $y + \dfrac{3}{16}\cdot\dfrac{1}{y} = 1,$ etc.

EXERCISE XCIV [a].   (PAGE 327.)

1. $4x^3 - 3x^2 + 2x - 1.$
     2. $8x^3 - 12x^2 + 6x - 1.$

3. $8x^3 - 12x^2y + 6xy^2 - y^3.$
     4. $x^3 - 2x^2y + 2xy^2 - y^3.$

5. $2 - 3x - x^2 + 2x^3.$
     6. $x^4 - 2x^3y + 3xy^3 - y^4.$

7. $1 + \frac{1}{2}a - \frac{1}{8}a^2 + \frac{1}{16}a^3 - \frac{5}{128}a^4.$

[b.]   (PAGE 328.)

1. $x^2 - x + 1.$
     2. $2 - 4x + x^2.$

3. $1 + 3x - x^2.$
     4. $y^2 - y + 2.$

5. $x^3 - x^2y + xy^2 + y^3.$
     6. $a^2 - \frac{1}{3}a^{-1} + \frac{1}{3}a^{-2}.$

7. $1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3.$

[c.]

1. $1\frac{3}{4}.$
   2. $1\frac{1}{2}.$
   3. $10.$
   4. $-\frac{3}{4}.$

5. $(a^4 - d) \div (c - 2a^3).$
   6. $b \div (a).$

7. 8.   8. 6.   9. $-\frac{1}{2}$.

10. $\pm 3a$.   11. $(m - n)^2 + a^2 = 0$.

12. Condition for square is $q^2 = 4p^2 \cdot (qr + q^2)$, etc.

13. See Ex. 3, p. 327; remainder in this case is $12ab^2x^2 - 24b^3$, which must $= 0$, etc.

14. $(a^{\frac{1}{3}}x + \frac{1}{3}a^{-\frac{2}{3}}b)^3 =$ given expression; expand and equate coefficients, $\frac{1}{3}a^{-1}b^2 = c$, $\frac{1}{27}a^{-2}b^3 = d$.